STOCHASTIC POPULATION MODELS

EXERCISES 4-5

4.
Consider the logistic equation as derived in section 1.9 of the lecture notes:

$$\begin{cases} \frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right) \\ r &= \frac{\alpha\beta e_0}{\delta}-\gamma \\ K &= e_0-\frac{\gamma\delta}{\alpha\beta} \end{cases}$$

(a) Give the transfer function $T(\omega)$, the gain $|T(\omega)|$ and the cutoff frequency ω_c for low-amplitude periodic fluctuations in the germination rate β.

(b) How are the above filter characteristics are affected by the other parameters of the model?

5.
Let P denote a reproductive plant, R a rosette (=pre-reproductive plant), S a seed and E an empty safe site, and consider the following processes:

$$\begin{array}{rcll} \mathrm{P} & \xrightarrow{\alpha} & \mathrm{S}+\mathrm{P} & \text{(seed production)} \\ \mathrm{S}+\mathrm{E} & \xrightarrow{\beta} & \mathrm{R} & \text{(colonization of a safe-site)} \\ \mathrm{P} & \xrightarrow{\gamma} & \mathrm{E} & \text{(plant death)} \\ \mathrm{S} & \xrightarrow{\delta} & \dagger & \text{(seed death)} \\ \mathrm{R} & \xrightarrow{p,\varphi} & \mathrm{P} & \text{(maturation)} \end{array}$$

where p is the *per capita* probability of surviving the rosette state and $\varphi(\tau)$ the probability density of the time τ that survivors spent as a rosette.

(a) Derive the corresponding population equations assuming that the rate of seed production and the death rate of seeds are very high compared to the colonization rate, the death rate of plants and rosettes and the rate of the transition from a rosette into a plant.

(For reference, see sections 1.9 and 3.1 of the lecture notes.)