SECOND COURSE IN STATISTICS 20.1.–28.4.2011. Literature: Sheldon M. Ross: Introductory Statistics. Lecturer: University Lecturer Pekka Pere.

Suggested answers to the 1 st set of exercises 27.1.2011

1. **Expected value** and **population mean** are synonymous concepts relating to populations, while **sample mean** $\overline{X}$ and **sample average** are synonymous concepts relating to samples from populations.

2. Given the following properties of expected value,

$$\mathsf{E}(cX_i) = c\mathsf{E}(X_i) = c\mu_i, \tag{1}$$

$$\mathsf{E}(c + X_i) = c + \mathsf{E}(X_i) = c + \mu_i, \tag{2}$$

$$\mathsf{E}(X_i + X_j) = \mathsf{E}(X_i) + \mathsf{E}(X_j) = \mu_i + \mu_j. \tag{3}$$

We can expand the equation:

$$\mathsf{E}\left[c + \sum_{i=1}^{k} a_i X_i\right] = \mathsf{E}\left[c + a_1X_1 + a_2X_2 + ... + a_kX_k\right]$$

Apply property (2):

$$\mathsf{E}\left[c + \sum_{i=1}^{k} a_i X_i\right] = c + \mathsf{E}\left[a_1X_1\right] + \mathsf{E}\left[a_2X_2\right] + ... + \mathsf{E}\left[a_kX_k\right]$$

Since all a_i are constants $(i = 1, ..., k)$, we can apply property (1):

$$\mathsf{E}\left[c + \sum_{i=1}^{k} a_i X_i\right] = c + a_1\mathsf{E}\left[X_1\right] + a_2\mathsf{E}\left[X_2\right] + ... + a_k\mathsf{E}\left[X_k\right]$$

Apply property (3):

$$\mathsf{E}\left[c + \sum_{i=1}^{k} a_i X_i\right] = c + a_1\mu_1 + a_2\mu_2 + ... + a_k\mu_k = c + \sum_{i=1}^{k} a_i\mu_i$$

3. Given the following properties of variance,

$$\mathsf{var}(cX_i) = c^2\mathsf{var}(X_i)) = c^2\sigma_i^2 \tag{4}$$

$$\mathsf{var}(c + X_i) = \mathsf{var}(X_i) = \sigma_i^2 \tag{5}$$

$$\mathsf{var}(\sum_{i=1}^{k} X_i) = \sum_{i=1}^{k} \mathsf{var}(X_i) = \sum_{i=1}^{k} \sigma_i^2 \tag{6}$$

A constant c never varies. So we start the proof by applying property (5):

$$\mathsf{var}\left[c + \sum_{i=1}^{k} a_i X_i\right] = \mathsf{var}\left[\sum_{i=1}^{k} a_i X_i\right]$$

We can then expand our equation:

$$\mathsf{var}\left[c + \sum_{i=1}^{k} a_i X_i\right] = \mathsf{var}\left[a_1 X_1 + a_2 X_2 + ... + a_k X_k\right]$$

Since all a_i are constants $(i = 1, ..., k)$, we can apply property (4):

$$\mathsf{var}\left[c + \sum_{i=1}^{k} a_i X_i\right] = a_1^2\mathsf{var}\left[X_1\right] + a_2^2\mathsf{var}\left[X_2\right] + ... + a_k^2\mathsf{var}\left[X_k\right]$$

$$\mathsf{var}\left[c + \sum_{i=1}^{k} a_i X_i\right] = a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + ... + a_k^2\sigma_k^2$$

And finally, resum the values as in property (6):

$$\mathsf{var}\left[c + \sum_{i=1}^{k} a_i X_i\right] = \sum_{i=1}^{k} a_i^2\sigma_i^2.$$

4.
a) Since we assume the random variables are independent, we can reapply the variance property (6) in the previous question:

$$\mathsf{var}\left[X_1 + X_2\right] = \sigma_1^2 + \sigma_2^2$$

However,

$$\mathsf{var}\left[X_1 - X_2\right] = \sigma_1^2 + \sigma_2^2$$

because we consider a negative value as a constant $c = -1$, and apply the variance property (4) such that:

$$\mathsf{var}\left[X_1\right] + \mathsf{var}\left[-X_2\right] = \mathsf{var}\left[X_1\right] + (-1)^2\mathsf{var}\left[X_2\right] = \sigma_1^2 + \sigma_2^2$$

b) When $X_1 = X_2$,

$$\mathsf{var}\left[X_1 + X_2\right] = \mathsf{var}\left[X_2 + X_2\right] = \mathsf{var}\left[2X_2\right] = 4\sigma_2^2$$

by variance property (4). And

$$\mathsf{var}\left[X_1 - X_2\right] = \mathsf{var}\left[X_2 - X_2\right] = \mathsf{var}\left[0\right] = 0.$$

5.
a) Since the probabilities 0.7 and 0.3 add up to 1, they represent all possible changes of the stock. Expected value of stock X can then be defined as:

$$\mathsf{E}(X) = 105 \times 0.7 + 75 \times 0.3 = 96$$

b) Yes, it is possible that the probability of the stock price rising is higher than the probability that it falls, but nevertheless the investor sells his stock. As shown in part a), if the amount that the price falls (here 25) is much greater than the amount that the price increases (here 5), then the *expected value* of the stock is less than the *market value* of the stock ($96 < 100$ euros). So even though there is a greater chance of gaining a small profit (of up to 5) with the stock, the chance of a huge loss (of up to 25) is strong enough to lower the expected value of the stock, and so the investor sells.

c) The potential amounts of return are on the X-axis. The Normal density function (Gaussian curve) depicts a bell shaped curve as a function of X. The area under the density function between two values on the X-axis gives the probability that the return takes a value in between the two values. In case of the Normal distribution the density function is symmetric and the expected return lies exactly below the top point of the density function. The correct explanation should therefore read "*... the expected return is the value on the X-axis corresponding to the point at the top of the density.*"