Dynamics of linear operators – Exercise set 5 (8. 12. 2010)

1. Let $S = \{z \in \mathbb{C} : |z| = 1\}$ be the unit circle and $f\colon S \to S$, $f(z) = e^{2\pi i\alpha} z$, a rotation with α irrational (see Exercise 1.2). Is f weakly mixing or even mixing?

2. Consider the double tent map defined in Exercise 1.6. Prove that it is not weakly mixing.

3. Suppose that X is a Banach space and $T \in L(X)$. Show that if T satisfies the Hypercyclicity criterion, $T \times T$ does too. (This completes the proof of Theorem 4.7.)

4. a) Let $D\colon H(\mathbb{C}) \to H(\mathbb{C})$, $Df = f'$, be MacLane's operator. We know that it is hypercyclic, even mixing (Example 4.6). Show that λD is mixing for all $\lambda \in \mathbb{C} \setminus \{0\}$.
   b) Suppose that X is a Banach space. Show that there is no operator $T \in L(X)$ such that λT would be hypercyclic for all $\lambda \in \mathbb{K} \setminus \{0\}$.

5. Suppose that X is a metric space which has at least one isolated point. Show that if $f\colon X \to X$ is continuous and transitive, then X is finite and f is bijective.

6. Let X be a metric space and $f\colon X \to X$ continuous. Prove that the following conditions are equivalent:

   i) f is weakly mixing.

   ii) For any nonempty open sets $U, V_1, V_2 \subset X$, there exists $n \geq 0$ such that
   $$f^n(U) \cap V_1 \neq \emptyset \quad \text{and} \quad f^n(U) \cap V_2 \neq \emptyset.$$

   iii) For any nonempty open sets $U, V \subset X$, there exists $n \geq 0$ such that
   $$f^n(U) \cap U \neq \emptyset \quad \text{and} \quad f^n(U) \cap V \neq \emptyset.$$

   [*Suggestion.* Argue that (iii) implies (ii) implies (i). The following fact might be needed: $f^{-k}(U)$ is nonempty and open for $k \geq 1$ whenever U is nonempty and open.]

Dictionary: (weakly) mixing = (heikosti) sekoittava, isolated point = erakkopiste

Note: No classes on Mon 6. 12.