**Effect of standardization of covariates**

In the normal linear model with uninformative prior, the posterior density of regression parameters becomes (conditionally to $\sigma$) the following multivariate normal density:

$$\pi(\beta \mid y, X, \sigma) = N\Big(\underbrace{(X^T X)^{-1} X^T y}_{\text{mean vector}}\,,\ \underbrace{(X^T X)^{-1}\sigma^2}_{\text{cov. matrix}}\Big).$$

The example of York rainfall data: using R, compute posterior means according to the above multivariate normal density, and compute the covariance matrix (assume $\sigma = 1$) under original $x$ variables, and under standardized variables $x - \bar{x}$:

```
y <- c(41,52,18.7,55,40,29.2,51,17.6,46.6,57)
x <- c(23.9,43.3,36.3,40.6,57,52.5,46.1,142,112.6,23.7)

X <- matrix(1,10,2)
for(i in 1:10){X[i,2]<-x[i]}
# posterior means for regression parameters beta:
betahat <- ((solve(t(X)%*%X))%*%t(X))%*%y
# covariance matrix of beta, assuming sigma=1:
C <- solve(t(X)%*%X)
```

This gives

$$\text{C} = \begin{bmatrix} 0.346762974 & -4.269256e-03 \\ -0.004269256 & 7.386255e-05 \end{bmatrix}$$

```
# the same with standardized x:
xs <- x-mean(x)

Xs <- matrix(1,10,2)
for(i in 1:10){Xs[i,2]<-xs[i]}
# posterior means for regression parameters beta:
betahats <- ((solve(t(Xs)%*%Xs))%*%t(Xs))%*%y
# covariance matrix of beta, assuming sigma=1:
Cs <- solve(t(Xs)%*%Xs)
```

This gives

$$\text{Cs} = \begin{bmatrix} 1.0000e-01 & -2.099300e-19 \\ -2.0993e-19 & 7.386255e-05 \end{bmatrix}$$

which is nearly a diagonal matrix, so that the correlations are now almost vanished. Hence, we can expect to obtain a posterior distribution of $\beta$ with minimal correlations. The parameters of the standardized model and the original model are related, because:

$$\mu_i = \beta_0^* + \beta_1^*(x_i - \bar{x}) = \underbrace{\beta_0^* - \beta_1^*\bar{x}}_{\beta_0} + \beta_1^* x_i = \beta_0 + \beta_1^* x_i$$

Hence, with this standardization, $\beta_j$ for $j \geq 1$ would remain the same, but $\beta_0$ of the original model would correspond to $\beta_0^* - \beta_1^* \bar{x}$ in the standardized model.

For curiosity, in BUGS, we could compute the same matrix, but since it is a function of data, it is a constant and cannot be monitored as an uncertain parameter. Its values can be checked using `Info` $\rightarrow$ `Node info` $\rightarrow$ `values`.

```
model{
for (i in 1:n) {
y[i] ~ dnorm(mu[i],tau.y)
xx[i] <- (x[i]-mean(x[]))
mu[i] <- a + b*xx[i] # standardized x
#mu[i] <- a + b*x[i]
X[i,1] <- 1
X[i,2] <- xx[i]
for(t in 1:2){XT[t,i] <- X[i,t] }
}
for(i in 1:2){for(j in 1:2){XTX[i,j] <- inprod(XT[i,],X[,j]) }}
XTXinv[1:2,1:2] <- inverse(XTX[,])
... ...
```