

Hierarchical models

- In the non-hierarchical models we simply had
 - Conditional density of data $\pi(y \mid \text{parameters})$.
 - Prior density of parameters $\pi(\text{parameters})$.
 - to get posterior $\pi(\text{parameters} \mid y)$.

Hierarchical models

- In the non-hierarchical models
 - As a default, the prior is often 'noninformative'.
 - Can expect posterior mode to resemble maximum likelihood estimate, since we basically "normalize the likelihood function" to become a probability density:
$$\pi(\text{parameters}|y) = \pi(y|\text{parameters}) \times 1 / \text{const.}$$
 - not utilizing the full potential of Bayesian models, unless we elicit informative priors?

Hierarchical models

- In the non-hierarchical models
 - Informative priors can be laborious to get, although may be essential ingredient in some problems. (e.g. expert opinions).
 - "Empirical Bayes": draw the informative priors from data
 - "Prior" data should be independent of the "actual" data, to avoid using same data twice.

Hierarchical models

- With hierarchical data (groups within groups)
 - Natural idea to borrow strength from rest of the groups, to support weak data in some groups → add hyperprior to the model to make it hierarchical.
 - Accomplishes the effect of informative prior, but over whole hierarchical structure, based on **conditional independency**.
 - extremely versatile approach to many “hard problems” where data are unbalanced, partially missing, etc, but where the total information can help “filling the gaps”.
 - Bayesian “information synthesis”.