

Table A.1 Continuous distributions

| Distribution | Notation | Parameters |
|---------------------------|--|---|
| Uniform | $\theta \sim U(a, b)$ $p(\theta) = U(\theta a, b)$ | boundaries a, b with $b > a$ |
| Normal | $\theta \sim N(\mu, \sigma^2)$ $p(\theta) = N(\theta \mu, \sigma^2)$ | location μ scale $\sigma > 0$ |
| Multivariate normal | $\theta \sim N(\mu, \Sigma)$ $p(\theta) = N(\theta \mu, \Sigma)$ (implicit dimension d) | symmetric, pos. definite, $d \times d$ variance matrix Σ |
| Gamma | $\theta \sim \text{Gamma}(\alpha, \beta)$ $p(\theta) = \text{Gamma}(\theta \alpha, \beta)$ | shape $\alpha > 0$ inverse scale $\beta > 0$ |
| Inverse-gamma | $\theta \sim \text{Inv-gamma}(\alpha, \beta)$ $p(\theta) = \text{Inv-gamma}(\theta \alpha, \beta)$ | shape $\alpha > 0$ scale $\beta > 0$ |
| Chi-square | $\theta \sim \chi^2_\nu$ $p(\theta) = \chi^2_\nu(\theta)$ | degrees of freedom $\nu > 0$ |
| Inverse-chi-square | $\theta \sim \text{Inv-}\chi^2_\nu$ $p(\theta) = \text{Inv-}\chi^2_\nu(\theta)$ | degrees of freedom $\nu > 0$ |
| Scaled inverse-chi-square | $\theta \sim \text{Inv-}\chi^2(\nu, s^2)$ $p(\theta) = \text{Inv-}\chi^2(\theta \nu, s^2)$ | degrees of freedom $\nu > 0$ scale $s > 0$ |
| Exponential | $\theta \sim \text{Expon}(\beta)$ $p(\theta) = \text{Expon}(\theta \beta)$ | inverse scale $\beta > 0$ |
| Wishart | $W \sim \text{Wishart}_\nu(S)$ $p(W) = \text{Wishart}_\nu(W S)$ (implicit dimension $k \times k$) | degrees of freedom ν symmetric, pos. definite $k \times k$ scale matrix S |
| Inverse-Wishart | $W \sim \text{Inv-Wishart}_\nu(S^{-1})$ $p(W) = \text{Inv-Wishart}_\nu(W S^{-1})$ (implicit dimension $k \times k$) | degrees of freedom ν symmetric, pos. definite $k \times k$ scale matrix S |

| Density function | Mean, variance, and mode |
|---|--|
| $p(\theta) = \frac{1}{b-a}, \theta \in [a, b]$ | $E(\theta) = \frac{a+b}{2}, \text{var}(\theta) = \frac{(b-a)^2}{12}$ no mode |
| $p(\theta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(\theta - \mu)^2\right)$ | $E(\theta) = \mu, \text{var}(\theta) = \sigma^2$ mode(θ) = μ |
| $p(\theta) = (2\pi)^{-d/2} \Sigma ^{-1/2} \times \exp\left(-\frac{1}{2}(\theta - \mu)^T \Sigma^{-1}(\theta - \mu)\right)$ | $E(\theta) = \mu, \text{var}(\theta) = \Sigma$ mode(θ) = μ |
| $p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta}, \theta > 0$ | $E(\theta) = \frac{\alpha}{\beta}$ $\text{var}(\theta) = \frac{\alpha}{\beta^2}$ mode(θ) = $\frac{\alpha-1}{\beta}$, for $\alpha \geq 1$ |
| $p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{-(\alpha+1)} e^{-\beta/\theta}, \theta > 0$ | $E(\theta) = \frac{\beta}{\alpha-1}$, for $\alpha > 1$ $\text{var}(\theta) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$, $\alpha > 2$ mode(θ) = $\frac{\beta}{\alpha+1}$ |
| $p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} \theta^{\nu/2-1} e^{-\theta/2}, \theta > 0$ same as Gamma($\alpha = \frac{\nu}{2}, \beta = \frac{1}{2}$) | $E(\theta) = \nu, \text{var}(\theta) = 2\nu$ mode(θ) = $\nu - 2$, for $\nu \geq 2$ |
| $p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} \theta^{-(\nu/2+1)} e^{-1/(2\theta)}, \theta > 0$ same as Inv-gamma($\alpha = \frac{\nu}{2}, \beta = \frac{1}{2}$) | $E(\theta) = \frac{1}{\nu-2}$, for $\nu > 2$ $\text{var}(\theta) = \frac{2}{(\nu-2)^2(\nu-4)}$, $\nu > 4$ mode(θ) = $\frac{1}{\nu+2}$ |
| $p(\theta) = \frac{(\nu/2)^{\nu/2}}{\Gamma(\nu/2)} s^\nu \theta^{-(\nu/2+1)} e^{-\nu s^2/(2\theta)}, \theta > 0$ same as Inv-gamma($\alpha = \frac{\nu}{2}, \beta = \frac{\nu}{2} s^2$) | $E(\theta) = \frac{\nu}{\nu-2} s^2$ $\text{var}(\theta) = \frac{2\nu^2}{(\nu-2)^2(\nu-4)} s^4$ mode(θ) = $\frac{\nu}{\nu+2} s^2$ |
| $p(\theta) = \beta e^{-\beta\theta}, \theta > 0$ same as Gamma($\alpha = 1, \beta$) | $E(\theta) = \frac{1}{\beta}, \text{var}(\theta) = \frac{1}{\beta^2}$ mode(θ) = 0 |
| $p(W) = \left(2^{\nu k/2} \pi^{k(k-1)/4} \prod_{i=1}^k \Gamma\left(\frac{\nu+1-i}{2}\right)\right)^{-1} \times S ^{-\nu/2} W ^{-(\nu-k-1)/2} \times \exp\left(-\frac{1}{2} \text{tr}(S^{-1}W)\right)$, W pos. definite | $E(W) = \nu S$ |
| $p(W) = \left(2^{\nu k/2} \pi^{k(k-1)/4} \prod_{i=1}^k \Gamma\left(\frac{\nu+1-i}{2}\right)\right)^{-1} \times S ^{\nu/2} W ^{-(\nu+k+1)/2} \times \exp\left(-\frac{1}{2} \text{tr}(SW^{-1})\right)$, W pos. definite | $E(W) = (\nu - k - 1)^{-1} S$ |

Table A.1 Continuous distributions continued

| Distribution | Notation | Parameters |
|--------------------------------|---|--|
| Student- <i>t</i> | $\theta \sim t_\nu(\mu, \sigma^2)$ $p(\theta) = t_\nu(\theta \mu, \sigma^2)$ t_ν is short for $t_\nu(0, 1)$ | degrees of freedom $\nu > 0$ location μ scale $\sigma > 0$ |
| Multivariate Student- <i>t</i> | $\theta \sim t_\nu(\mu, \Sigma)$ $p(\theta) = t_\nu(\theta \mu, \Sigma)$ (implicit dimension d) | degrees of freedom $\nu > 0$ location $\mu = (\mu_1, \dots, \mu_d)$ symmetric, pos. definite $d \times d$ scale matrix Σ |
| Beta | $\theta \sim \text{Beta}(\alpha, \beta)$ $p(\theta) = \text{Beta}(\theta \alpha, \beta)$ | 'prior sample sizes' $\alpha > 0, \beta > 0$ |
| Dirichlet | $\theta \sim \text{Dirichlet}(\alpha_1, \dots, \alpha_k)$ $p(\theta) = \text{Dirichlet}(\theta \alpha_1, \dots, \alpha_k)$ | 'prior sample sizes' $\alpha_j > 0; \alpha_0 \equiv \sum_{j=1}^k \alpha_j$ |

Table A.2 Discrete distributions

| Distribution | Notation | Parameters |
|-------------------|---|---|
| Poisson | $\theta \sim \text{Poisson}(\lambda)$ $p(\theta) = \text{Poisson}(\theta \lambda)$ | 'rate' $\lambda > 0$ |
| Binomial | $\theta \sim \text{Bin}(n, p)$ $p(\theta) = \text{Bin}(\theta n, p)$ | 'sample size' n (positive integer) 'probability' $p \in [0, 1]$ |
| Multinomial | $\theta \sim \text{Multin}(n; p_1, \dots, p_k)$ $p(\theta) = \text{Multin}(\theta n; p_1, \dots, p_k)$ | 'sample size' n (positive integer) 'probabilities' $p_j \in [0, 1]; \sum_{j=1}^k p_j = 1$ |
| Negative binomial | $\theta \sim \text{Neg-bin}(\alpha, \beta)$ $p(\theta) = \text{Neg-bin}(\theta \alpha, \beta)$ | shape $\alpha > 0$ inverse scale $\beta > 0$ |
| Beta-binomial | $\theta \sim \text{Beta-bin}(n, \alpha, \beta)$ $p(\theta) = \text{Beta-bin}(\theta n, \alpha, \beta)$ | 'sample size' n (positive integer) 'prior sample sizes' $\alpha > 0, \beta > 0$ |

Density function

Mean, variance, and mode

| | |
|--|--|
| $p(\theta) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\pi\sigma^2}} (1 + \frac{1}{\nu}(\frac{\theta-\mu}{\sigma})^2)^{-(\nu+1)/2}$ | $E(\theta) = \mu, \text{ for } \nu > 1$ $\text{var}(\theta) = \frac{\nu}{\nu-2}\sigma^2, \text{ for } \nu > 2$ $\text{mode}(\theta) = \mu$ |
| $p(\theta) = \frac{\Gamma((\nu+d)/2)}{\Gamma(\nu/2)\nu^{d/2}\pi^{d/2}} \Sigma ^{-1/2} \times (1 + \frac{1}{\nu}(\theta - \mu)^T \Sigma^{-1} (\theta - \mu))^{-(\nu+d)/2}$ | $E(\theta) = \mu, \text{ for } \nu > 1$ $\text{var}(\theta) = \frac{\nu}{\nu-2}\Sigma, \text{ for } \nu > 2$ $\text{mode}(\theta) = \mu$ |
| $p(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}$ $\theta \in [0, 1]$ | $E(\theta) = \frac{\alpha}{\alpha+\beta}$ $\text{var}(\theta) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ $\text{mode}(\theta) = \frac{\alpha-1}{\alpha+\beta-2}$ |
| $p(\theta) = \frac{\Gamma(\alpha_1+\dots+\alpha_k)}{\Gamma(\alpha_1)\dots\Gamma(\alpha_k)} \theta^{\alpha_1-1} \dots \theta^{\alpha_k-1}$ $\theta_1, \dots, \theta_k \geq 0; \sum_{j=1}^k \theta_j = 1$ | $E(\theta_j) = \frac{\alpha_j}{\alpha_0}$ $\text{var}(\theta_j) = \frac{\alpha_j(\alpha_0-\alpha_j)}{\alpha_0^2(\alpha_0+1)}$ $\text{cov}(\theta_i, \theta_j) = -\frac{\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)}$ $\text{mode}(\theta_j) = \frac{\alpha_j-1}{\alpha_0-k}$ |
| Density function | Mean, variance, and mode |
| $p(\theta) = \frac{1}{\theta!} \lambda^\theta \exp(-\lambda)$ $\theta = 0, 1, 2, \dots$ | $E(\theta) = \lambda, \text{var}(\theta) = \lambda$ $\text{mode}(\theta) = \lfloor \lambda \rfloor$ |
| $p(\theta) = \binom{n}{\theta} p^\theta (1-p)^{n-\theta}$ $\theta = 0, 1, 2, \dots, n$ | $E(\theta) = np$ $\text{var}(\theta) = np(1-p)$ $\text{mode}(\theta) = \lfloor (n+1)p \rfloor$ |
| $p(\theta) = \binom{n}{\theta_1, \theta_2, \dots, \theta_k} p_1^{\theta_1} \dots p_k^{\theta_k}$ $\theta_j = 0, 1, 2, \dots, n; \sum_{j=1}^k \theta_j = n$ | $E(\theta_j) = np_j$ $\text{var}(\theta_j) = np_j(1-p_j)$ $\text{cov}(\theta_i, \theta_j) = -np_i p_j$ |
| $p(\theta) = \binom{\theta+\alpha-1}{\alpha-1} \left(\frac{\beta}{\beta+1}\right)^\alpha \left(\frac{1}{\beta+1}\right)^\theta$ $\theta = 0, 1, 2, \dots$ | $E(\theta) = \frac{\alpha}{\beta}$ $\text{var}(\theta) = \frac{\alpha}{\beta^2}(\beta+1)$ |
| $p(\theta) = \frac{\Gamma(n+1)}{\Gamma(\theta+1)\Gamma(n-\theta+1)} \frac{\Gamma(\alpha+\theta)\Gamma(n+\theta-\theta)}{\Gamma(\alpha+\theta)\Gamma(n-\theta+1)}$ $\times \frac{\Gamma(\alpha+\theta)}{\Gamma(\alpha)\Gamma(\theta)}, \theta = 0, 1, 2, \dots, n$ | $E(\theta) = n \frac{\alpha}{\alpha+\beta}$ $\text{var}(\theta) = n \frac{\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}$ |