Exercise 8
This is an exercise in numerical methods. The aim of the exercise is to learn how to produce a pairwise invadability plot (PIP) if $s_x(y)$ is known but otherwise too complicated to do calculations by hand. Especially if you cannot solve this exercise, you should come to the exercise class where Paolo or Jaakko will teach you the required necessary methods.

Consider the invasion fitness $s_x(y)$ of the strategy $y$ in a monomorphic resident population of strategy $x$ in the Lotka-Volterra competition model as we have seen earlier in the course:

$$s_x(y) = r(y) \left(1 - \frac{a(y, x)K(x)}{K(y)}\right)$$  \hspace{1cm} (1)

(a) Produce PIPs (numerically!) if $K(x) = e^{-x^2}$ and $a(x, y) = e^{-\alpha(x-y)^2}$ for different values of $\alpha > 0$ and compare the results with those in the lectures.

(b) If you managed the above, try some other forms of the functions $K$ and $a$ and interpret the results in terms of ecology (i.e., resident population dynamics) and in terms of the strategy dynamics. For example, try

$$K(x) = e^{-(x-\delta)^4} + e^{-(x+\delta)^2}$$  \hspace{1cm} (2)

$$a(x, y) = e^{-\alpha(x-y)^2} - \beta(x-y)$$  \hspace{1cm} (3)

for $\alpha = 2.0$, $\beta = -0.4$ and $\delta = 1.0$

(c) Make a plot of the singular strategies versus one of the model parameters. Can you tell which singular strategies are monomorphically attracting and which are not? Same question, but now for dimorphic attraction?

Exercise 9
Consider the following resident dynamics

$$\frac{\dot{n}_i}{n_i} = \frac{r(x_i) \sum_j n_j}{1 + \sum_j n_j} - 1 - \sum_j a(x_i, x_j)n_j \quad (i = 1, \ldots, k)$$  \hspace{1cm} (4)

with $r(x) = x$ and $a(x, y) = e^{x-y}$ (from Gyllenberg & Parvinen (2001), also see lecture notes).

(a) How does the resident population equilibrium depend on the resident strategy $x$?

(b) Produce a PIP and compare the results with those from the lecture notes.