Evolution in a dimorphic resident population

(→ Geritz et al. 1998)

Readily generalizes to polymorphic population.

Low mutation rate
("mutation-limited evolution")

→ Only one mutant at a time

So, we can study the possible evolutionary change for each resident strategy separately, pretending the other resident strategy is fixed.

This is how that goes:

First, let $P_2$ denote the set of all strategy pairs $(x_1, x_2)$ that can coexist.

Note that $P_2$ is not necessarily the same as the set of mutually invading strategies, although in L.V. models this happens to be true.
Evolutionary isocline

The $x_i$-isocline ($i=1,2$) is the set
\[
\{(x_1, x_2) \in \mathbb{R}_2 \mid \left[ \frac{\partial}{\partial y} s_{x_1, x_2}(y) \right]_{y=x_i} = 0 \}\]

In other words, the $x_i$-isocline in the collection of all points $(x_1, x_2)$ where the selection gradient $\left[ \frac{\partial}{\partial y} s_{x_1, x_2}(y) \right]_{y=x_i}$ (in the $x_i$-direction) vanishes.

In different words again, $(x_1, x_2)$ lies on the $x_i$-isocline if $x_i$ is singular with respect to evolution in $x_i$ if $x_j (j \neq i)$ were kept constant.

Example

(a)

(b) $P_2$ (dark region)

PIP for $x_2$ assuming that $x_1$ stays fixed
If the points on an isocline can be interpreted as "singular points" (in at least one of the strategies) they can be classified in a similar way (see "2 cases")

Suppose \( \frac{\partial}{\partial y} S_{x,x_i}(y) \bigg|_{y=x_i} = 0 \), i.e., \((x,x_i)\) in a point on the \( x_i \)-isocline.

\[ \frac{\partial^2}{\partial y \partial y} S_{x,x_i}(y) \bigg|_{y=x_i} < 0 \Rightarrow (x_1,x_2) \text{ is unavoidable for (small) mutations in } x_i \text{ (but not necessarily in } x_j, j \neq i \).

\[ \frac{\partial^2}{\partial y \partial y} S_{x,x_i}(y) \bigg|_{y=x_i} < \frac{\partial^2}{\partial x_2 \partial x_2} S_{x,x_2}(y) \bigg|_{y=x_i} \Rightarrow (x_1,x_2) \text{ is attracting for evolution in the } x_i \text{-direction (but not necessarily in the } x_j \text{ (} j \neq i \text{) direction).}

\text{etc.}

If \((x_1,x_2)\) lies both on the \( x_i \)-isocline and on the \( x_2 \)-isocline, we call \((x_1,x_2)\) a singular point.