Malliteoria

Harjoitus 6

1. Show that there are $\mathcal{A}$ and $\mathcal{B}$ such that they are elementarily equivalent but II does not win $EF_1(\mathcal{A},\mathcal{B})$.

2. Let $L = \{S\}$, S a unary function symbol. For $1 < n < \omega$, let $\mathcal{A}_n$ be an L-structure such that $dom(\mathcal{A}_n) = \{0, ..., n\}$ and $S^{\mathcal{A}_n}(x) = x + 1$ if $x < n$ and otherwise 0. Let $\mathcal{A}_\omega$ be such that $dom(\mathcal{A}_\omega) = \mathbf{Z}$ and $S^{\mathcal{A}_\omega}(x) = x + 1$ for all $x \in \mathbf{Z}$. Show that if $n \geq 2^{k+1}$, then $II \uparrow EF_k(\mathcal{A}_\omega, \mathcal{A}_n)$.

3. Let L be as above. Find L-structures $\mathcal{A}$ and $\mathcal{B}$ such that they are elementarily equivalent but there is $a \in \mathcal{A}$ such that $a \notin dom(f)$ for any partial isomorphism $f : \mathcal{A} \rightarrow \mathcal{B}$.

4. Show that for all $n < \omega$, $EF_n(\mathcal{A},\mathcal{B})$ is determined i.e. if I does not have a winning strategy for $EF_n(\mathcal{A},\mathcal{B})$ (this is defined as for II), then $II \uparrow EF_n(\mathcal{A},\mathcal{B})$.