Malliteoria

Harjoitus 5

1. Let L consist of unary predicates P_i, $i < 4$, binary predicates R, Q, S and T and constants c_i, $i < \omega$. Let T_e be the theory that says:

(a) P_i, $i < 4$, is a partition of the universe, for all $i < \omega$, $P_0(c_i)$ and for all $i < j < \omega$, $c_i \neq c_j$,

(b) $\forall x \forall y (R(x,y) \rightarrow P_0(x) \wedge P_1(y))$, $\forall x \forall y (Q(x,y) \rightarrow P_2(x) \wedge P_1(y))$, $\forall x \forall y ($ $S(x,y) \rightarrow P_0(x) \wedge P_3(y))$ and $\forall x \forall y (T(x,y) \rightarrow P_1(x) \wedge P_3(y))$,

(c) for all $i < \omega$, $\forall y (R(c_i, y) \rightarrow \forall x (P_2(x) \rightarrow Q(x,y)))$,

(d) for all $i < \omega$, $\forall x (S(c_i, x) \leftrightarrow \forall y (R(c_i, y) \rightarrow T(y,x)))$.

Show that if $\mathcal{A}$ is an existentially closed model of T_e, then

$$\mathcal{A} \models \forall y (\forall x (P_2(x) \rightarrow Q(x,y)) \leftrightarrow \bigvee_{i<\omega} R(c_i, y))$$

and

$$\mathcal{A} \models \forall x (\bigwedge_{i<\omega} S(c_i, x) \leftrightarrow \forall y (\forall z (P_2(z) \rightarrow Q(z,y)) \rightarrow T(y,x))).$$

2. Let T_e be as above. Show that it has AP.

3. Let $L = \{<\}$, $<$ is a 2-ary predicate symbols, and let T_{lo} (lo for linear ordering) consist of the following sentences:

$\forall v_0 \forall v_1 \forall v_2 ((v_0 < v_1 \wedge v_1 < v_2) \rightarrow v_0 < v_2)$

$\forall v_0 \forall v_1 (v_0 < v_1 \rightarrow \neg v_1 < v_0)$

$\forall v_0 \forall v_1 (v_0 < v_1 \vee v_0 = v_1 \vee v_1 < v_0)$.

Show that T_{lo} has AP, JEP and is closed under unions.

4. Let T_{lo} be as above. Find a theory T so that the models of T are exactly the existentially closed models of T_{lo}.