Malliteoria

Harjoitus 2

1. Show that there exists $R \subseteq \mathbf{R}^2$ such that for all countable $X, Y \subseteq \mathbf{R}$, if $X \cap Y = \emptyset$, then there is $z \in \mathbf{R}$ such that $(z, x) \in R$ for all $x \in X$ and $(z, y) \notin R$ for all $y \in Y$.
Fact: $|\{X \subseteq \mathbf{R} |\ |X| \leq \omega\}| = |\mathbf{R}| = 2^\omega$.

2. Show that there are no $\{f\}$-theory T, $\#f = 1$, such that $\mathcal{A} = (\mathcal{A}, f) \models T$ iff for all $x \in \mathcal{A}$ there is $n \in \mathbb{N} - \{0\}$ such that $f^n(x) = x$, where $f^0(x) = x$ and $f^{n+1}(x) = f(f^n(x))$.

In the rest of the exercises, $U \subseteq \mathcal{P}(\omega)$ is such an ultrafilter that for all $n \in \omega$, $\{x \in \omega |\ x \geq n\} \in U$.

3. Suppose that for all $i \in \omega$, $\mathcal{A}_i$ is a finite structure. Show that $|\Pi_{i\in\omega}\mathcal{A}_i/U| \neq \omega$.

4. For all $i \in \omega$, let $\mathcal{A}_i = (\mathbf{Q}, <)$. Show that there is $f : (\mathbf{R}, <) \to \Pi_{i\in\omega}\mathcal{A}_i/U$ such that for all $x, y \in \mathbf{R}$, $x < y$ iff $f(x) < f(y)$.

5. Suppose that for all $n \in \omega$, $\mathcal{A}_n = (n + 1, <)$. Show that $\Pi_{i\in\omega}\mathcal{A}_i/U$ is not a well-ordering.