Minimal surfaces, H-systems and regularity.

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Abstract

Minimal surfaces are commonly known as soap films or soap bubbles. Mathematically we have a closed curve in $\mathbb{R}^3$ and we are looking for a surface with the smallest possible area, spanned on that curve - i.e. having it as its boundary. This problem was stated in the 18^{th} century by Joseph-Louis de Lagrange, who actually discovered differential equations describing the surface, but was not able to solve them. Next, in the middle of the 19^{th} century, came the famous experiments of Plateau, who popularized the problem and whose name became inseparably linked with it.

In my talk I will present the modern way of thinking about minimal surfaces and about so-called *surfaces of prescribed mean curvature*. There are several different approaches to this problem. In a variational setting, one tries to minimize a functional calculating the area of the surface, which leads to Euler-Lagrange equations. Other approach is to employ the fact that a minimal surface has to have constant mean curvature (denoted by H) in each point, which leads to a different system of differential equations. Then, we can generalize the problem by making H a function $H(x)$ and ask if there are surfaces with mean curvature at each point given by H. It turns out that these surfaces are also minimal in appropriate sense.

All these approaches lead to systems of second order partial differential equations. These systems have so-called *weak solutions*, the existence of which is guaranteed by some abstract methods from functional analysis. A priori, weak solutions may be even discontinuous, so a problem of regularity arises. Are the solutions smooth, differentiable, continuous? I will briefly discuss known results and describe the methods used to show regularity.