The dimension of projected measures on Riemann manifolds

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The dimensional properties of projections of sets and measures have been studied for decades and they are quite well understood. For example for the Hausdorff dimension we have the following theorem: If μ is a compactly supported Radon measure on $\mathbb{R}^n$, then for almost all $V \in G(n,m)$

$$\dim_{\mathrm{H}} P_{V*}\mu = \dim_{\mathrm{H}} \mu \quad \text{provided that} \quad \dim_{\mathrm{H}} \mu \leq m.$$

On the other hand, if $\dim_{\mathrm{H}} \mu > m$, then for almost all $V \in G(n,m)$

$$P_{V*}\mu \ll \mathcal{H}^m|_V.$$

These theorems are quite often "almost all"-results, which give no information about any specific projection. However, in 2003 Ledrappier and Lindenstrauss discovered that similar methods work also for one specific projection. They studied measures on the unit tangent bundle SM of a Riemann surface M and they proved the following theorem: Let μ be a Radon probability measure on SM. If μ is invariant under the geodesic flow, then

$$\dim_{\mathrm{H}} \Pi_*\mu = \dim_{\mathrm{H}} \mu, \text{ if } \dim_{\mathrm{H}} \mu \leq 2, \quad \text{and} \quad \Pi_*\mu \ll \mathcal{H}^2|_M, \text{ if } \dim_{\mathrm{H}} \mu > 2.$$

Here $\Pi : SM \to M$ is the natural projection. Järvenpää, Järvenpää and Leikas reproved the above theorem using the generalized projection formalism introduced by Peres and Schlag. Their proof also explains why the dimension of μ is not necessarily preserved if the dimension of the manifold M is greater than 2.

We will consider their proof and show that their methods can also be applied to prove similar projection theorems for the packing dimension and the q-dimension for $1 < q \leq 2$.