INVARIANT SUBSPACE PROBLEM

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"Does every bounded linear operator T on a separable infinite dimensional complex Hilbert space $\mathcal{H}$ have a non-trivial invariant subspace?" Here non-trivial invariant subspace means a closed linear subspace $M \subset \mathcal{H}$ such that

$$\{0\} \neq M \neq \mathcal{H} \quad \text{and} \quad Tx \in M \quad \text{for all} \quad x \in M.$$

This problem, even though very easy to state, has been open for several decades. In the talk some results concerning the same question in other types of spaces are presented. Also some partial results for the orginal question and for some related questions are given.

Topics include (at least):

(1) Answer to this question in the cases with $\dim \mathcal{H} < \infty$ and in the case of non-separable Hilbert spaces.
(2) Operator without any eigenvectors.
(3) Some classes of operators for which the answer is known to be positive.
(4) Every operator has a non-trivial invariant linear subspace, which might not be closed.