CONTINUITY AND DIFFERENTIABILITY OF ORLICZ-SOBOLEV FUNCTIONS

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This talk may be considered to continue the talk "Orlicz spaces as a generalization of L^p spaces" given by Sauli Lindberg in this seminar. We are interested in Orlicz-Sobolev spaces as a generalization of Sobolev spaces.

Let Ω be an open subset of $\mathbb{R}^n$ with $n \geq 2$. For $1 \leq p < \infty$ a function u belongs to the Sobolev space $W^{1,p}(\Omega)$ if and only if u and its weak partial derivatives belong to $L^p(\Omega)$. It is known that if $p > n$, then $u \in W^{1,p}_{\text{loc}}(\Omega)$ is continuous (at least by modifying u on a set of measure zero). Further, u is differentiable almost everywhere. Instead, if $p \leq n$, then these results are no longer true.

The Orlicz-Sobolev space $W^{1,\Phi}(\Omega)$ is defined as the set of those functions in the Orlicz space $L^\Phi(\Omega)$ whose weak partial derivatives belong to $L^\Phi(\Omega)$. For the definition of Orlicz spaces, we refer to the abstract of Sauli's talk. Above mentioned results concerning Sobolev spaces can be generalized for the Orlicz-Sobolev functions. We give a simple sufficient and necessary condition so that the Orlicz-Sobolev function $u \in W^{1,\Phi}_{\text{loc}}(\Omega)$ is continuous and almost everywhere differentiable.

As an example we study the case when the function Φ is defined by $\Phi(t) = t^n(\log(e + t))^\theta$ with $\theta \in [0, \infty)$.

The talk will be in english.