

# Application of statistics in engineering

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## 4. Application of multivariate factor analysis (FA)

### 4.1. Introduction to FA

Factor analysis is a statistical method designed to transfer a set of intercorrelated variables into a new set of uncorrelated components or factors, which account for the most of variance in the original variables. This technique is used in data reduction to explain the pattern of intercorrelations among variables. One may use FA to generate hypotheses regarding causal mechanisms or to screen variables for subsequent statistical analysis (for example regression analysis). It should be mentioned that FA is generally applied in social sciences, but

also find various applications in data processing in engineering, physics, chemistry etc.

In FA the original observed variables are modelled as linear combinations of the factors, plus some "error" terms in the form

$$\mathbf{X} = \mathbf{FL} + \mathbf{E} \quad (1)$$

where  $\mathbf{X} \in \mathbb{R}^{n \times m}$  is the matrix of the original intercorrelated data values,  $\mathbf{F} \in \mathbb{R}^{n \times k}$  is the matrix of factor scores, calculated for every respondent,  $\mathbf{L} \in \mathbb{R}^{k \times m}$  is the matrix of the factor loadings, and  $\mathbf{E} \in \mathbb{R}^{n \times m}$  is the error matrix. In (1) the number of factors  $k$  should be selected by the researcher.

In common, FA is based on a reduced correlation matrix. After obtaining of the matrix  $\mathbf{F}$  (extraction of factors or initial solution), it is recommended to carry out the so called rotation of factors. According to the method of rotation the rotated

solution represents geometrically the orthogonal or oblique directions in the all amount of data.

It must be noted that FA is not very well formalized statistical technique and makes sense in the case of suitable interpretation of the obtained results.

## 4.1. General assumptions in FA

- ◆ Data could show a random character. If necessary, some randomization procedure can be used.
- ◆ The recommended number of respondent values for each variable could be at least 50 or more.
- ◆ The variables should be quantitative at the interval or ratio level. Categorical data are not admitted for factor analysis.
- ◆ In the ideal case, the data should have a bivariate normal distribution for each pair of variables, and observations can be independent.
- ◆ The totality of data must satisfy special tests of adequacy, such as KMO, Bartlett's test, etc.

## 4.2. SPSS statistics

For each FA one can calculate:

- the correlation matrix of variables, with corresponding significance levels,
- determinant,
- inverse and reproduced correlation matrix, including anti-image,
- initial solution (communalities, eigenvalues, and percentage of variance explained),
- Kaiser-Meyer-Olkin measure of sampling adequacy and Bartlett's test of sphericity,
- unrotated solution, communalities, and eigenvalues,
- rotated solution with factor loadings, for oblique rotations: rotated pattern and structure matrices,
- factor scores

### 4.3. The main procedure of FA

- ◆ Calculation of the correlation matrix
- ◆ Verification of the tests for adequacy of FA
- ◆ Application of Principal Component Analysis (PCA) and/or other technique to obtain the communalities and total data variances
- ◆ Choosing the number of factors
- ◆ Extraction of factors
- ◆ Rotation of factors and obtaining the factor loadings
- ◆ Calculation and saving of the factor scores for consecutive study.

## **4.4. How to carry out FA in SPSS?**

- 1) Enter your data.** They will be automatically transformed into z-variables (with mean =0 and standard deviation = 1).
- 2) From the main menu of SPSS choose:**

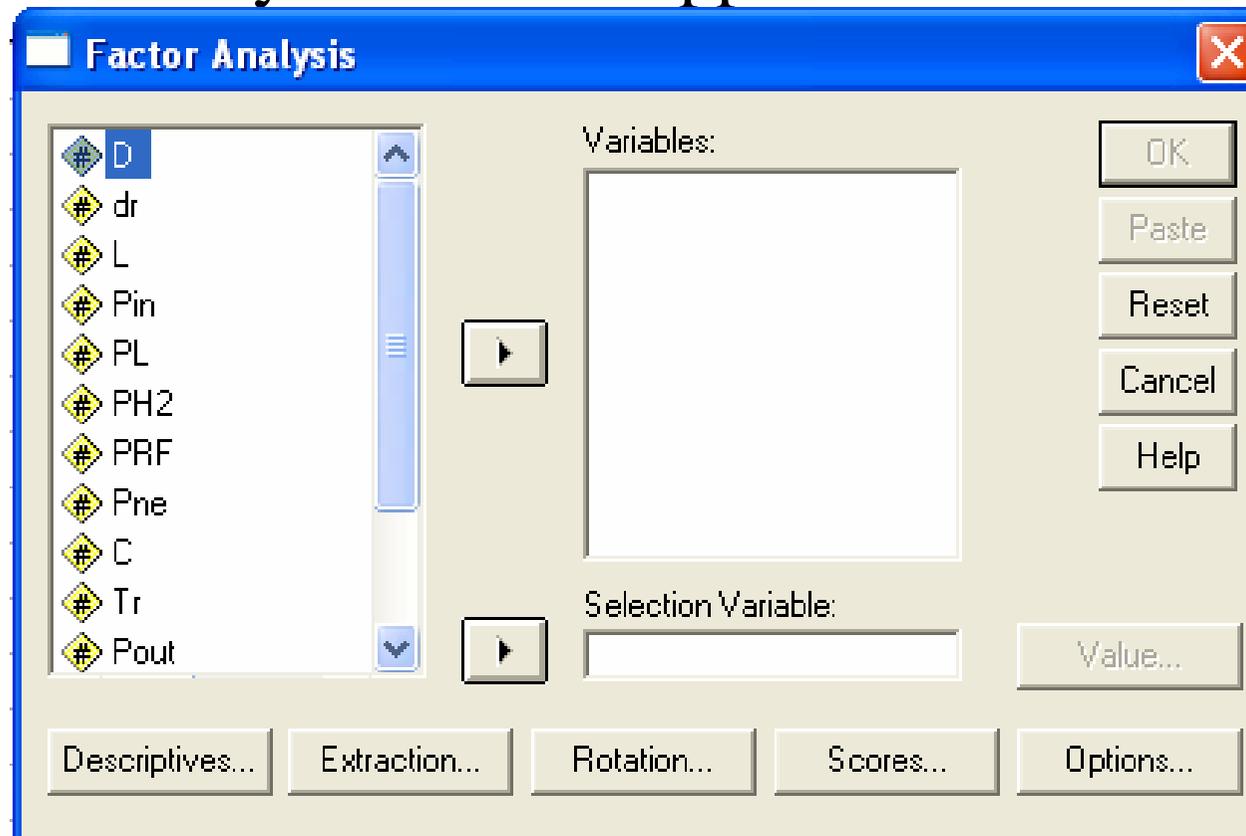
# Analyze/Data reduction/Factor

The screenshot shows the SPSS Data Editor window for a file named '157 s dobavki - Greece.sav'. The 'Analyze' menu is open, and the 'Data Reduction' option is selected, which has opened a sub-menu where 'Factor...' is highlighted. The data table in the background contains the following information:

	D	dr	PL	PH2	PRF	Pne	C	Tr	F
1	50,00	4,5	1,70	,35	20,00	100,00	,47	480,00	
2	15,00	4,5	1,70	,35	27,00	50,00	,47	480,00	
3	46,00	20,0	1,20	,30	125,50	87,00	1,10	485,00	
4	46,00	20,0	1,20	,30	15,50	15,00	1,10	418,00	
5	15,00	4,5	1,70	,35	20,00	80,00	,47	480,00	
6	15,00	4,5	1,70	,35	21,00	50,00	,47	480,00	
7	46,00	20,0	1,20	,96	15,50	15,00	1,10	490,00	
8	15,00	4,5	1,70	,35	35,00	50,00	,47	480,00	
9	15,00	4,5	1,70	,35	40,00	50,00	,47	480,00	
10	15,00	4,5	1,70	,35	45,00	50,00	,47	480,00	
11	15,00	4,5	1,70	,35	20,00	18,00	,47	480,00	
12	15,00	4,5	1,70	,35	52,00	50,00	,47	480,00	
13	46,00	20,0	1,20	,30	125,50	62,00	1,10	485,00	
14	40,00	20,0	1,20	,02	15,50	15,00	1,10	490,00	
15	40,00	20,0	1,20	,80	15,50	15,00	1,10	490,00	
16	15,00	4,5	1,70	,35	20,00	40,00	,47	480,00	
17	15,00	4,5	1,70	,35	20,00	18,00	,47	480,00	
18	15,00	4,5	1,70	,35	20,00	40,00	,47	480,00	

The status bar at the bottom indicates 'SPSS Processor is ready'.

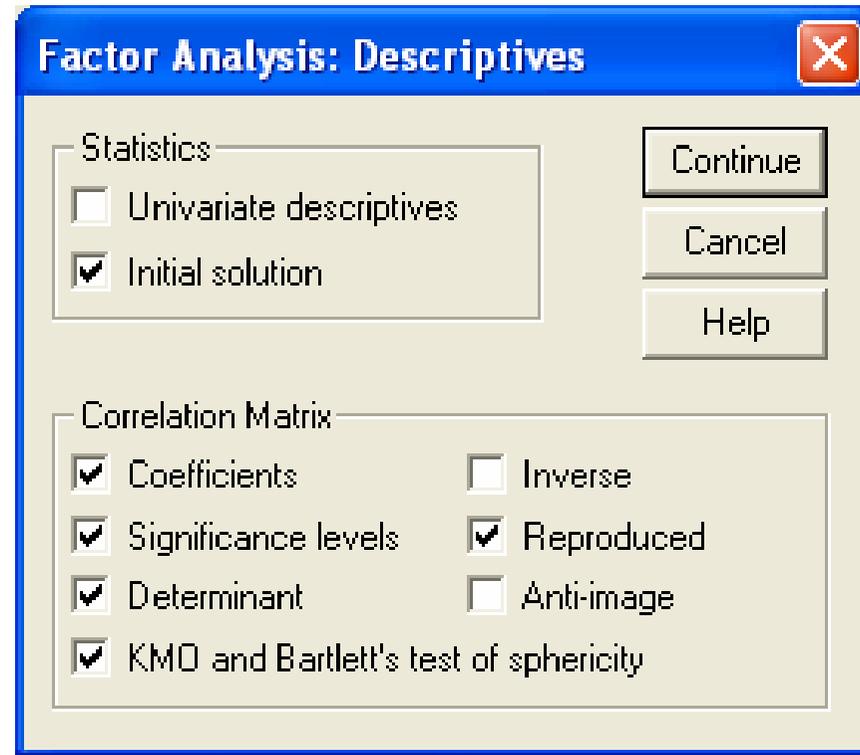
The main Factor analysis window appears:



**3) Move desired variables from the left to the area Variables in the right.** At the initial stage the dependent variable(s) can also be included, to check the strengths of correlations between all the

variables concerned with the study. At the consecutive stages the dependent variables should be omitted.

4) Select the options from  + **Continue**.

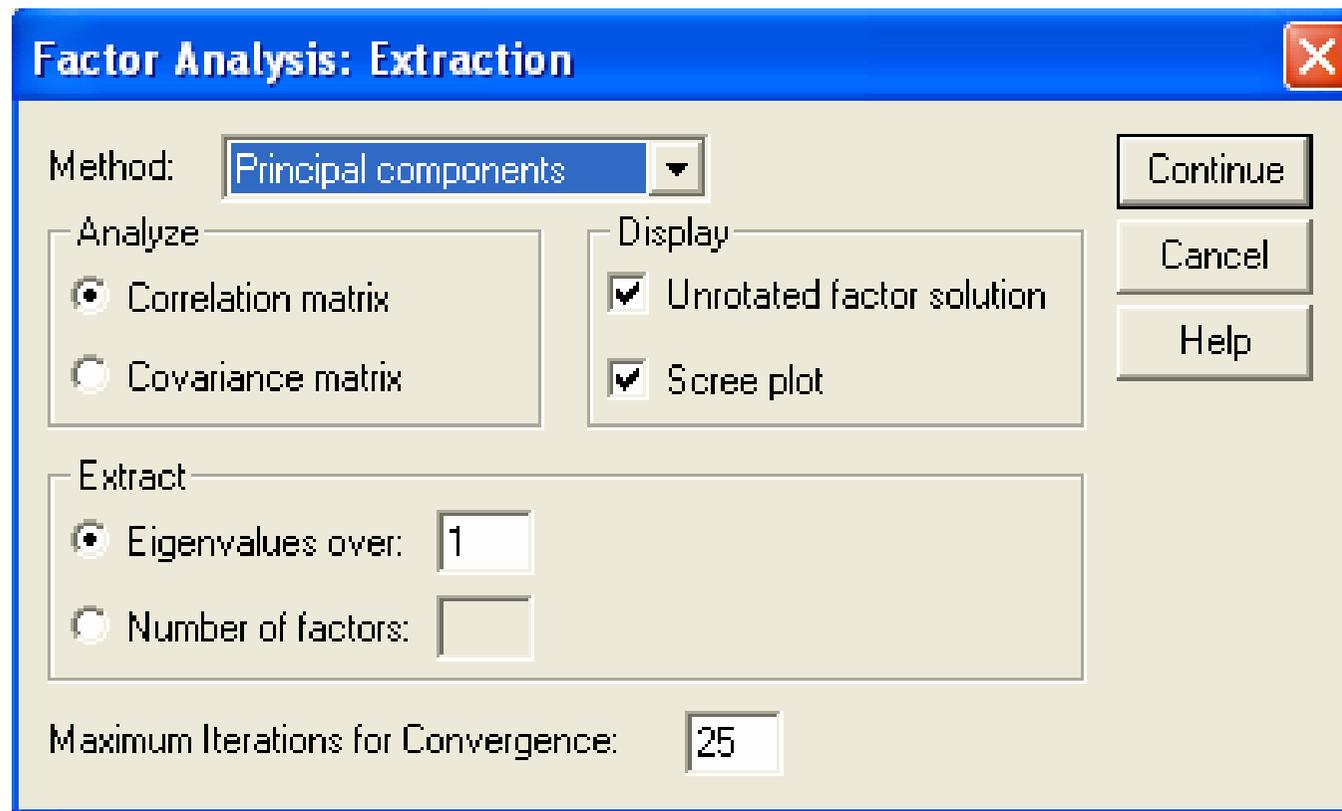


We are interested to find out the variables which correlate each another and this correlations are statistically significant, i.e. the

correlation coefficients must be at least  $>0,3$  and higher, with corresponding Sig. level  $<0,05$ . If a variable exists that correlates strongly the dependent variable(s), but does not correlate other independent variables, this variable is called unique variable. All unique variables must be removed prior to FA and could be added later in regression or other type of statistical study.

The FA is a good idea under some conditions of its adequacy. In SPSS one have to obtain the values of KMO measure of sampling adequacy  $>0,5$  and Bartlett's test for checking the sphericity in data, or Sig.  $<0,5$ .

5) From the Factor Analysis window open . Choose the method of extraction, the number of factors and the results to display, and validate by Continue.

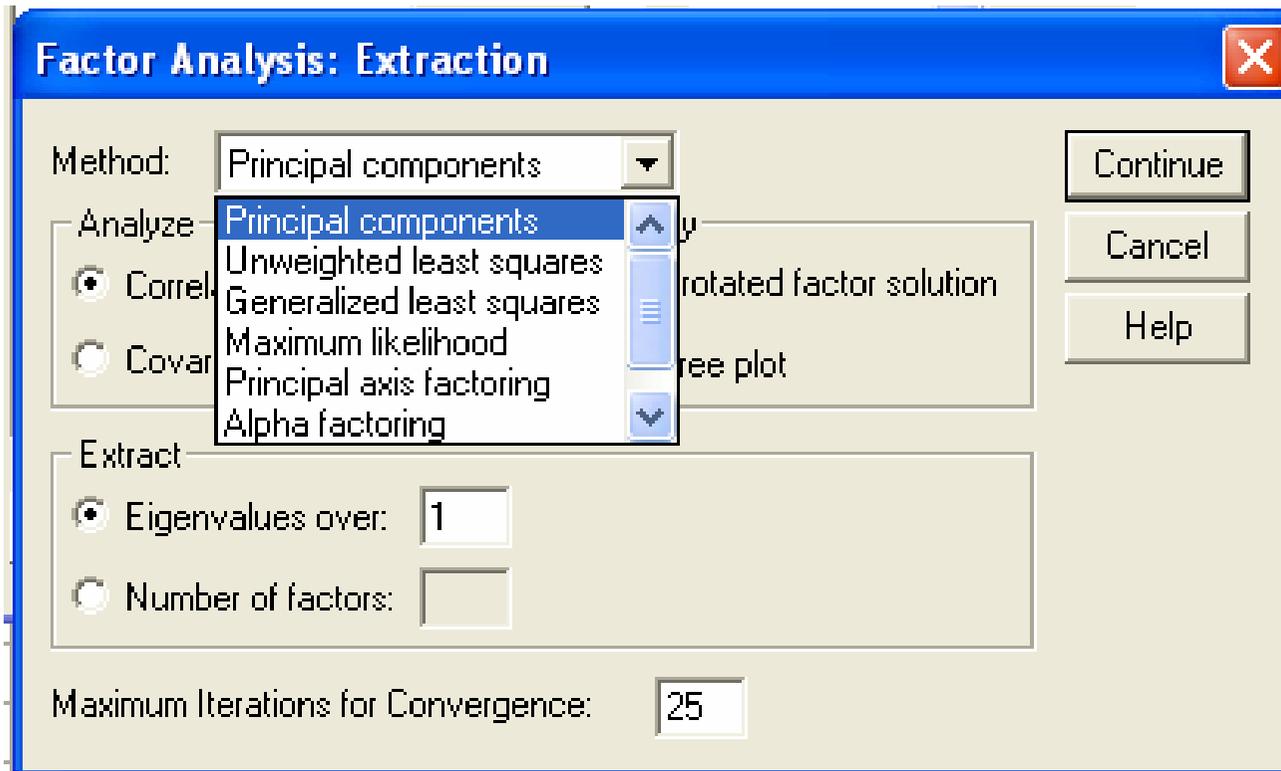


SPSS provides 7 different methods for the factor extraction:

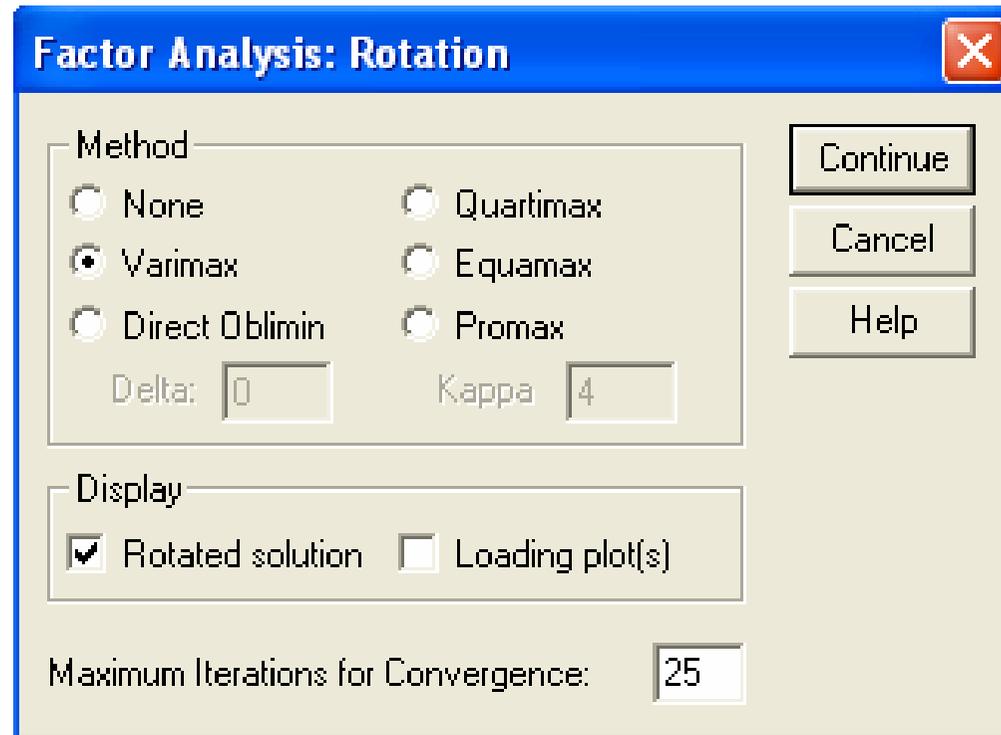
- ◆ Principal component analysis (default method)
- ◆ Unweighted least squares
- ◆ Generalized least squares
- ◆ Maximum likelihood
- ◆ Principal axis factoring
- ◆ Alpha factoring
- ◆ Image factoring

Usually one starts with the Principal components. This method obtains  $n$  factors from  $n$  variables with their relative weights in the total variance of data. So the sum of variances in PCA is always equal to 1.

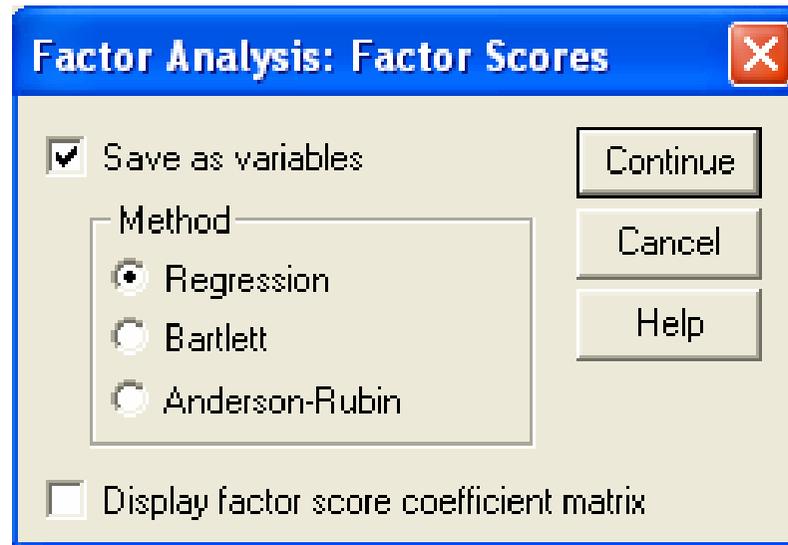
The number of factors depends on the absolute magnitudes of the eigenvalues of the correlation matrix. Usually, the number of factors is set to the number of eigenvalues greater than 1, but an option to choose other number is also available.



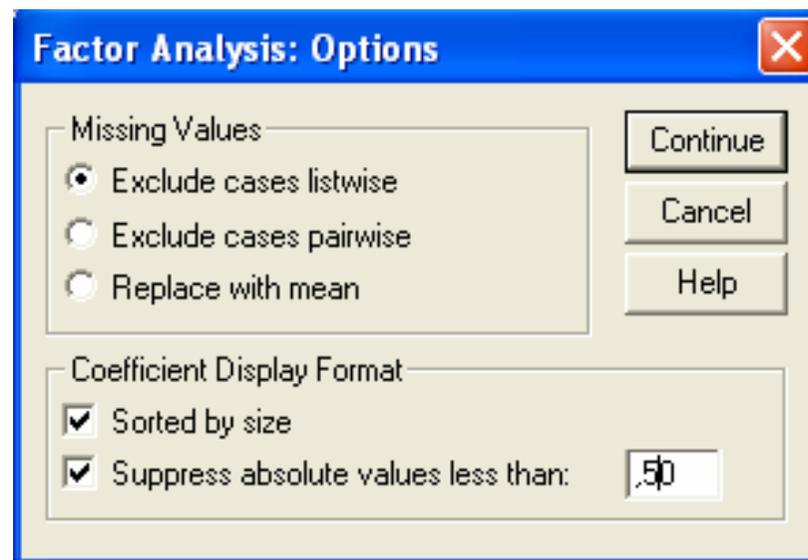
**6) Select the method of rotation of the factors by  window and validate by **Continue**.** There are 6 methods for rotation of factors. The most usual is the Varimax method. Mathematically the rotation of factors means a change of variables in order to obtain their mutual orthogonality or some oblique positions (pseudo-orthogonality).



**7) Choose to save calculated factor scores as variables by the  button. It is recommended to activate this option when the previous steps in FA procedure are obtained.**



**8) In Factor analysis/Options** one may set the preferred missing value algorithm and the format of coefficients



**9) In Factor analysis window select OK button to start the analysis.**

The results will appear in a separate window and could be saved as a separate output file.

## **5. Factor analysis for examining copper bromide laser parameters**

As an example we will present some results of FA and RA for the data of metal vapor lasers. This is a retrospective study, based on experiments, obtained in the period of the last 3 decades. It must be noted that the new laser device costs about 100000 Euro and other type of statistical study is very problematical to perform.

**5.1. Subject of study:** Low-temperature impulse copper bromide laser, from the group of metal-vapor lasers, characterized by [1-3]:

- ◆ wavelengths 510.6 nm and 578.2 nm,
- ◆ maximum output power up to 500 W,
- ◆ the most efficient and produce the highest output power in the visible region.

- ◆ The laser finds wide application in medicine, chemistry, for the investigation of the atmosphere, for aerial and submarine location and navigation, in modern micro and nanotechnologies, etc.

The basic input parameters, which characterize the CuBr laser functionality are: technological laser design (geometry and constructive materials), operational conditions (input electric power, gas pressure, metal vapor pressure, pulse repetition rate, different additives, etc.). The major output parameters are: the generated output laser power  $P_{out}$  and the overall laser efficiency  $Eff$ .

In further examples we use the different random samples from original data, obtained in the Metal Vapour Lasers Department, Georgi Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences. They are published basically in [4]-[11].

## 5.2. Data selection

A 25% random sample from 300 experiments of 12 parameters:

- $D$ ,  $mm$  – inside diameter of the laser tube,
- $d_r$ ,  $mm$  – inside diameter of the rings,
- $L$ ,  $cm$  – length of the active area (electrode separation),
- $P_{in}$ ,  $kW$  – input electrical power,
- $PL$ ,  $kW/m$  – input electrical power per unit length,
- $P_{H_2}$ ,  $Torr$  – hydrogen gas pressure,
- $P_{rf}$ ,  $kHz$  – pulse repetition rate,  $P_{ne}$ ,  $Torr$  - neon gas pressure,
- $C$ ,  $pF$  – equivalent capacity of the capacitor bank,
- $T_r$ ,  $^{\circ}C$  – temperature of the CuBr reservoirs,
- $P_{out}$ ,  $W$  – output laser power
- $Eff$ ,  $\%$  – laser efficiency

## 5.3. Results from the FA

### 1) Calculation of correlation matrices

TABLE 1. Correlation matrix <sup>a)</sup>

	Variable	D	dr	L	Pin	PL	PH2	Pfr	Pne	C	Tr	Pout	Eff
Correlation	D	1.000	.836	.749	.694	-.558	.294	-.089	-.186	.215	.113	.723	.651
	dr	.836	1.000	.916	.862	-.533	.411	-.169	-.211	.190	.221	.915	.815
	L	.749	.916	1.000	.855	-.697	.539	-.197	-.127	.086	.124	.914	.839
	Pin	.694	.862	.855	1.000	-.296	.372	-.170	-.095	.179	.116	.954	.699
	PL	-.558	-.533	-.697	-.296	1.000	-.493	.132	.310	-.065	-.004	-.426	-.621
	PH2	.294	.411	.539	.372	-.493	1.000	-.236	-.062	-.188	-.304	.469	.690
	Prf	-.089	-.169	-.197	-.170	.132	-.236	1.000	.262	-.059	.087	-.209	-.277
	Pne	-.186	-.211	-.127	-.095	.310	-.062	.262	1.000	-.307	.041	-.124	-.260
	C	.215	.190	.086	.179	-.065	-.188	-.059	-.307	1.000	.292	.129	.081
	Tr	.113	.221	.124	.116	-.004	-.304	.087	.041	.292	1.000	.100	-.055
	Pout	.723	.915	.914	.954	-.426	.469	-.209	-.124	.129	.100	1.000	.856
	Eff	.651	.815	.839	.699	-.621	.690	-.277	-.260	.081	-.055	.856	1.000

<b>Sig. (1- tai led)</b>	<b>D</b>		<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.005</b>	<b>.225</b>	<b>.056</b>	<b>.033</b>	<b>.169</b>	<b>.000</b>	<b>.000</b>
	<b>dr</b>	<b>.000</b>		<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.075</b>	<b>.036</b>	<b>.053</b>	<b>.029</b>	<b>.000</b>	<b>.000</b>
	<b>L</b>	<b>.000</b>	<b>.000</b>		<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.046</b>	<b>.140</b>	<b>.234</b>	<b>.147</b>	<b>.000</b>	<b>.000</b>
	<b>Pin</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>		<b>.005</b>	<b>.001</b>	<b>.074</b>	<b>.209</b>	<b>.063</b>	<b>.163</b>	<b>.000</b>	<b>.000</b>
	<b>PL</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.005</b>		<b>.000</b>	<b>.131</b>	<b>.004</b>	<b>.291</b>	<b>.486</b>	<b>.000</b>	<b>.000</b>
	<b>PH2</b>	<b>.005</b>	<b>.000</b>	<b>.000</b>	<b>.001</b>	<b>.000</b>		<b>.022</b>	<b>.299</b>	<b>.054</b>	<b>.004</b>	<b>.000</b>	<b>.000</b>
	<b>Prf</b>	<b>.225</b>	<b>.075</b>	<b>.046</b>	<b>.074</b>	<b>.131</b>	<b>.022</b>		<b>.016</b>	<b>.318</b>	<b>.242</b>	<b>.045</b>	<b>.012</b>
	<b>Pne</b>	<b>.056</b>	<b>.036</b>	<b>.140</b>	<b>.209</b>	<b>.004</b>	<b>.299</b>	<b>.016</b>		<b>.006</b>	<b>.370</b>	<b>.158</b>	<b>.017</b>
	<b>C</b>	<b>.033</b>	<b>.053</b>	<b>.234</b>	<b>.063</b>	<b>.291</b>	<b>.054</b>	<b>.318</b>	<b>.006</b>		<b>.008</b>	<b>.148</b>	<b>.256</b>
	<b>Tr</b>	<b>.169</b>	<b>.029</b>	<b>.147</b>	<b>.163</b>	<b>.486</b>	<b>.004</b>	<b>.242</b>	<b>.370</b>	<b>.008</b>		<b>.211</b>	<b>.329</b>
	<b>Pout</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.045</b>	<b>.158</b>	<b>.148</b>	<b>.211</b>		<b>.000</b>
	<b>Eff</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.012</b>	<b>.017</b>	<b>.256</b>	<b>.329</b>	<b>.000</b>	

**a) Determinant = 3.45E-007**

## **Analysis from the correlation matrix**

A correlation coefficient less than 0.3 indicate to a lack of significant relation among the corresponding variables. It is observed that only the first six independent variables have strong correlation with the dependent variables Pout and Eff and with each another. We may exclude the following four parameters prior to factor analysis: Prf, Pne, C and Tr, whose all correlation coefficients are less than 0.3. It must be also mentioned, that some statistical levels of significance of these variables are greater than 0.05 which means that their reliability is concerned with the error level of the experiments. So these four parameters are not acceptable for statistical analysis.

Table 1.2. Correlation Matrix <sup>(a)</sup> of six variables

		<b>D</b>	<b>dr</b>	<b>L</b>	<b>Pin</b>	<b>PL</b>	<b>PH2</b>
<b>Correlation</b>	<b>D</b>	<b>1.000</b>	<b>.836</b>	<b>.749</b>	<b>.694</b>	<b>-.558</b>	<b>.294</b>
	<b>dr</b>	<b>.836</b>	<b>1.000</b>	<b>.916</b>	<b>.862</b>	<b>-.533</b>	<b>.411</b>
	<b>L</b>	<b>.749</b>	<b>.916</b>	<b>1.000</b>	<b>.855</b>	<b>-.697</b>	<b>.539</b>
	<b>Pin</b>	<b>.694</b>	<b>.862</b>	<b>.855</b>	<b>1.000</b>	<b>-.296</b>	<b>.372</b>
	<b>PL</b>	<b>-.558</b>	<b>-.533</b>	<b>-.697</b>	<b>-.296</b>	<b>1.000</b>	<b>-.493</b>
	<b>PH2</b>	<b>.294</b>	<b>.411</b>	<b>.539</b>	<b>.372</b>	<b>-.493</b>	<b>1.000</b>
<b>Sig. (1-tailed)</b>	<b>D</b>		<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.005</b>
	<b>dr</b>	<b>.000</b>		<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>
	<b>L</b>	<b>.000</b>	<b>.000</b>		<b>.000</b>	<b>.000</b>	<b>.000</b>
	<b>Pin</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>		<b>.005</b>	<b>.001</b>
	<b>PL</b>	<b>.000</b>	<b>.000</b>	<b>.000</b>	<b>.005</b>		<b>.000</b>
	<b>PH2</b>	<b>.005</b>	<b>.000</b>	<b>.000</b>	<b>.001</b>	<b>.000</b>	

a) Determinant = .001

## 2) Verification of the tests for adequacy of FA

Table KMO and Bartlett's Tests

Test		For all data	For all 10 independent variables	For 6 independent variables
Kaiser-Meyer-Olkin Measure of Sampling Adequacy		.642	.518	.639
Bartlett's Test of Sphericity	Approx. Chi-Square	910.138	521.111	487.041
	df	66	45	15
	Sig.	.000	.000	.000

We found KMO=0.642, 0.518 and 0.639, so greater than 0.50 for all twelve variables, for 10 independent variables and for the first six variables, respectively. Also the two Bartlett's tests of sphericity are Sig.=0.000 This means that FA is applicable for our data.

### **3) Application of PCA and Principal Axis Factoring techniques to obtain the communalities and total data variances**

Next, the communalities and total variances for the first six variables by the Principal Component Analysis (PCA) and Principal Axis Factoring technique were carried out. The obtained results from the two methods were very similar.

Three factors were retained in order to better distinguish the influence of the parameters. In Table 2 variances for the three first eigenvalues by the PCA are given. Analysis of the reproduced correlations was also performed which showed satisfactory values of the residuals.

We can conclude that three factors will explain the total variance of 94.034% of our sample data.

## Table Communalities

	Initial	Extraction
D	1.000	.849
dr	1.000	.945
L	1.000	.944
Pin	1.000	.955
PL	1.000	.976
PH2	1.000	.973

Extraction Method: Principal Component Analysis (PCA).

Table 2. Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	4.127	68.787	68.787	4.127	68.787	68.787
2	.916	15.262	84.050	.916	15.262	84.050
3	.599	9.984	94.034	.599	9.984	94.034
4	.254	4.231	98.265			
5	.084	1.402	99.667			
6	.020	.333	100.000			

Extraction Method: Principal Component Analysis.

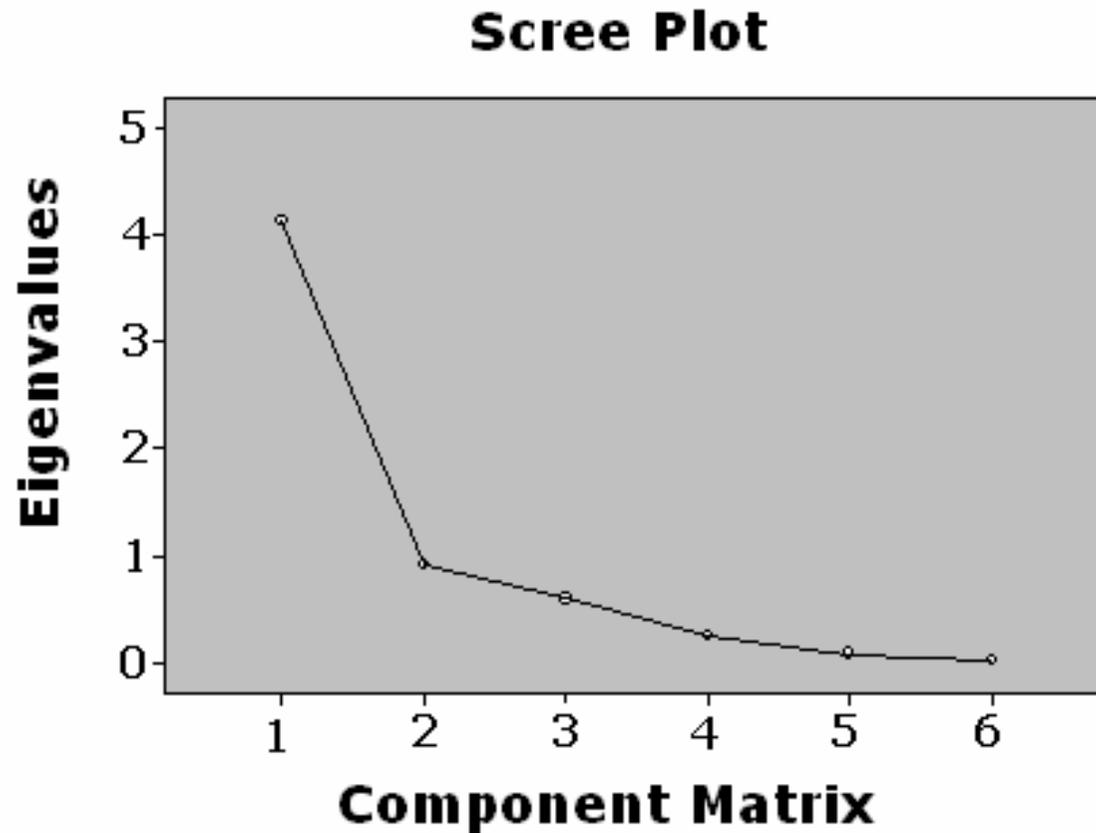
TABLE 2.2. Total variance explained for the first six variables.  
Extraction method: principal component analysis.

The last three columns represent the corresponding rotation sums of squared loadings for the Varimax rotation method.

Component	Initial Eigenvalues			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	4.127	68.787	68.787	3.088	51.460	51.460
2	0.916	15.262	84.050	1.388	23.138	74.598
3	0.599	9.984	94.034	1.166	19.436	94.034

## 4) Choosing the number of factors

Three factors were retained in order to better distinguish the influence of the parameters.



## 5) Extraction of factors

Table Component Matrix(a)

	Component		
	1	2	3
L	.971		
dr	.945		
D	.855		
Pin	.852		
PL	-.703	.501	
PH2	.584	-.649	

Extraction Method: Principal Component Analysis.

a) 3 components extracted.

**Remark.** The loadings less than 0,05 are omitted.

## **6) Rotation of factors and obtaining the factor loadings**

In this step we conducted the extraction of factors, following by the next methods of rotation: Varimax, Equamax, Quartimax, Oblimin and Promax, provided by SPSS. The more appropriate solutions were found via the Varimax and Promax methods. These results are given in Table 3. The almost equal values and order by magnitudes are valid for the two solutions. The corresponding rotation sums of squared loadings do not depend on the rotation method. Their values are given in Table 2. and Table 2.2.

TABLE 3. Rotated component matrix for Varimax and Promax rotation methods with Kaiser normalization. Extraction method: principal component analysis. The absolute values less than 0,5 are suppressed.

Variable	Components for the Varimax method			Components for the Promax method		
	1	2	3	1	2	3
<b>D</b>	<b>.792</b>			<b>.776</b>		
<b>dr</b>	<b>.906</b>			<b>.935</b>		
<b>L</b>	<b>.804</b>			<b>.751</b>		
<b>Pin</b>	<b>.950</b>			<b>1.086</b>		
<b>PL</b>		<b>-.918</b>			<b>-.985</b>	
<b>PH2</b>			<b>.941</b>			<b>.944</b>

**Analysis from the factor extraction procedure:**

In Tables 3 and 2 we can observe that the variables Pin, dr, L and D in this order (for the case of standard Varimax method), have high loadings on the first factor  $F_1$  with common variance of

51.461%. The variable PL loads high and negatively on the second factor  $F_2$  with variance 23.138%. The variable PH2 loads high on the third factor  $F_3$  with variance 19.436%.

## **7) Obtaining and saving the factor scores as new variables**

FA is performed

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## 6. Application of the results from FA in mulvariate regression analyses

In this section we apply the technique of multiple linear regression analysis to obtain the explicit form of the dependences between factors and output laser power Pout. Then the same procedure was carried out for Eff. The basic model statistics are given in Table 4.

TABLE 4. Model summaries for the two basic dependant variables Pout and Eff.

<b>Dependant variable</b>	<b>Regression Method</b>	<b>R</b>	<b>R Square</b>	<b>Adjusted R Square</b>	<b>Std. Error of the Estimate</b>	<b>Durbin-Watson</b>
<b>Pout</b>	<b>Linear</b>	<b>.970</b>	<b>.941</b>	<b>.938</b>	<b>3.97598</b>	<b>1.710</b>
<b>Eff</b>	<b>Linear</b>	<b>.897</b>	<b>.804</b>	<b>.796</b>	<b>.17677</b>	<b>1.228</b>

The most important here is the value of the R square index. In the case of Pout we can conclude that the obtained model describes

94.1% of our data and for Eff the correspondent model is valid for 80.4% of data. These results are very satisfactory. The calculated regression models for Pout by the factors from the varimax method in non standardized and standardized (dimensionless) forms are respectively

$$\begin{aligned}
 Pout &= 29.236F_1 + 3.786F_2 + 9.767F_3 + 33.594 \\
 \hat{Pout} &= 0.913\hat{F}_1 + 0.118\hat{F}_2 + 0.305\hat{F}_3
 \end{aligned} \tag{2}$$

For the laser efficiency Eff by using the same factors  $F_i$ ,  $i = 1,2,3$  we have

$$\begin{aligned}
 Eff &= 0.500F_1 + 0.282F_2 + 0.403F_3 + 1.508 \\
 \hat{Eff} &= 0.609\hat{F}_1 + 0.361\hat{F}_2 + 0.403\hat{F}_3
 \end{aligned} \tag{3}$$

## 7. Physical interpretation of the results

On the basis of the performed factor analysis we can classify the partial contributions of the initial laser parameters to the entire laser functionality. This type of information cannot be achieved by other methods.

The main results can be summarized as follows:

- ◆ the greatest value is that of the input power  $P_{in}$ , followed by the parameters of the geometrical design: the inside diameter of the rings  $d_r$ , the length of the active area  $L$  and the inside diameter of the laser tube,  $D$ . All these four parameters could be considered in a group. The higher electrical power  $P_{in}$  (with respectively lower  $PL$ ) enhances the energy of the electrons and the effectiveness of the upper laser level population. This process takes the central role in increasing of  $P_{out}$ . The rise of  $L$  explains the rise of

the amplification. The bigger inside diameter  $D$  improves the heat balance of the laser tube and the decrease of the heat population of the lower laser levels. The bigger rings improve the discharge stability.

- ◆ By the regression models (2), in particular from a standardized representation, we see that the first factor (i.e.  $P_{in}$ ,  $d_r$ ,  $L$  and  $D$ ) plays a dominant role in enhancement of the output laser power  $P_{out}$ . The parameters  $P_L$  and  $P_{H2}$  are of minor importance. For the efficiency  $Eff$ , the regression coefficient of the first factor is also dominant, but less than in  $P_{out}$ . In this case the increase of  $P_{in}$  leads to a saturation of  $P_{out}$ , which grows in lower degree.

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