TCM315 Fall 2022: Introduction to Open Quantum Systems

Lecture 17: General stochastic Schrödinger equation with jumps

Course handouts are designed as a study aid and are not meant to replace the recommended textbooks. Handouts may contain typos and/or errors. The students are encouraged to verify the information contained within and to report any issue to the lecturer.

CONTENTS

1. INTRODUCTION

These notes cover material expounded in § 6.1-6.3 of [3]. The theory of piecewise deterministic processes is reviewed in § 1.5-1.6 of [3]. The research paper [1] also offers an alternative self-contained introduction to quantum trajectories drive by counting processes.

2. GENERAL FORM IN A FINITE DIMENSIONAL HILBERT SPACE

The general form of a stochastic Schrödinger equation with jumps is

$$\mathrm{d}\boldsymbol{\psi}_t = -\mathrm{d}t \left(\imath\, \mathbb{H} + \frac{1}{2} \sum_{a=1}^{\mathscr{A}} \left(\mathbb{A}_a^\dagger\, \mathbb{A}_a - \|\mathbb{A}_a \boldsymbol{\psi}_t\|^2 \right) \right) \boldsymbol{\psi}_t + \sum_{a=1}^{\mathscr{A}} \mathrm{d}\nu_{a,t} \left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\, \boldsymbol{\psi}_t\|} - \mathbb{1} \right) \boldsymbol{\psi}_t \tag{1}$$

The source of stochasticity is the collection $\{\mathrm{d}\nu_{a,t}\}_{a=1}^{\mathscr{A}}$ of increments of **independent Poisson processes**. A Poisson process $\{\nu_t\}_{t\geq 0}$ is often referred to as a "counting process" as it takes values on natural numbers. In the present case $\{\nu_{a,t}\}_{t\geq 0}$ counts the number of quantum jumps associated to the detection of the a-th outcome.

The stochastic differentials in (1) must be understood as defined according to the **Itô (pre-point) discretization rule**. This means that the state vector ψ_t is independent of the $\{\mathrm{d}\nu_{a,t}\}_{a=1}^{\mathscr{A}}$. The characterization of the stochastic increments is complete by stating that the increments of the Poisson processes satisfy

$$\mathrm{d}\nu_{a,t}\mathrm{d}\nu_{a',t} = \delta_{aa'}\mathrm{d}\nu_{a',t} \tag{2}$$

i.e. they are **indepedent** for different values of the label a.

The jump rates are specified by the conditional expectation

$$\mathrm{E}\left(\mathrm{d}\nu_{a,t}|\boldsymbol{\psi}_t\right) = \|\mathbb{A}_a \boldsymbol{\psi}_t\|^2 \,\mathrm{d}t$$

We recall that (2) means that

1. with probability one, at time t only one jump can occur;
2. the values of the increments can only be $0, 1$

	$\mathrm{d}t$	$\mathrm{d}\nu_{a,t}$	$\mathrm{d}\nu_{b,t}$
$\mathrm{d}t$	0	0	0
$\mathrm{d}\nu_{a,t}$	0	$\mathrm{d}\nu_{a,t}$	0
$\mathrm{d}\nu_{b,t}$	0	0	$\mathrm{d}\nu_{b,t}$

TABLE I: Differential table for $\mathrm{d}\nu_{a,t}$, $\mathrm{d}\nu_{b,t}$ with $a \neq b$. This means that the two increments cannot take the unit value at the same time.

The label a ranges over the number $\mathscr{A}$ of distinct outcomes of a generalized measurement. In general, such number does not bear any necessary relation with the dimension of the Hilbert space. The operation $\mathbb{A}_a$ specifies the state-vector "collapse" occasioned by the occurrence of the a-th outcome of the generalized measurement. A relation between $\mathscr{A}$ and the dimension of the Hilbert space comes about if the $\{\mathbb{A}_a\}_{a=1}^{\mathscr{A}}$ coincide with the projectors onto the eigenspaces of a non-degenerate self-adjoint operator. In such a case, and when the Hilbert space $\mathcal{H}$ is finite dimensional $\mathcal{H} = \mathbb{C}^d$ the identity $\mathscr{A} = d$ holds true.

2.1. Compact representation of the equation

We may gain some insight by couching (1) into a more compact form. For this reason we define the *dissipative*, complex, Hamiltonian

$$\mathbb{K} := \imath\,\mathbb{H} + \frac{1}{2}\sum_{a=1}^{\mathscr{A}} \mathbb{A}_a^{\dagger}\mathbb{A}_a \tag{3}$$

We then notice that

$$\mathrm{Re}\langle \boldsymbol{\psi}\,, \mathbb{K}\boldsymbol{\psi}\rangle = \langle \boldsymbol{\psi}\,, \mathrm{Re}\,\mathbb{K}\boldsymbol{\psi}\rangle = \frac{1}{2}\sum_{a=1}^{\mathscr{A}}\langle \boldsymbol{\psi}\,, \mathbb{A}_a^{\dagger}\mathbb{A}_a\boldsymbol{\psi}\rangle = \frac{1}{2}\sum_{a=1}^{\mathscr{A}}\|\mathbb{A}_a\boldsymbol{\psi}\|^2 \tag{4}$$

We use (3) to write (1) as

$$\mathrm{d}\boldsymbol{\psi}_t = -\mathrm{d}t\left(\mathbb{K} - \mathrm{Re}\langle \boldsymbol{\psi}_t\,, \mathbb{K}\boldsymbol{\psi}_t\rangle\right)\boldsymbol{\psi}_t + \sum_{a=1}^{\mathscr{A}}\mathrm{d}\nu_{a,t}\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\boldsymbol{\psi}_t\|} - \mathbb{1}\right)\boldsymbol{\psi}_t \tag{5}$$

2.2. Preservation of the norm

A very noticeable difference between the Schrödinger equation and the stochastic Schrödinger equation (1) is the non-linearity of the latter. Non-linearity is needed to preserve the normalization of the state vector. Namely, stochastic Schrödinger equation preserves **pathwise** the normalization of the state vector.

Proposition. *For any state vector ψ_t such that $\|\psi_t\|^2 = 1$, the stochastic Schrödinger equation (1) implies*

$$\mathrm{d}\,\|\boldsymbol{\psi}_t\|^2 = 0$$

Proof.
The rule of stochastic differentiation

$$\mathrm{d}(fg) = f\mathrm{d}g + g\mathrm{d}f + (\mathrm{d}f)(\mathrm{d}g)$$

once applied to the norm of the state vector becomes

$$\mathrm{d}\|\boldsymbol{\psi}_t\|^2 = \langle \boldsymbol{\psi}_t\,, \mathrm{d}\boldsymbol{\psi}_t\rangle + \langle \mathrm{d}\boldsymbol{\psi}_t\,, \boldsymbol{\psi}_t\rangle + \langle \mathrm{d}\boldsymbol{\psi}_t\,, \mathrm{d}\boldsymbol{\psi}_t\rangle \;= 2\;\mathrm{Re}\langle \boldsymbol{\psi}_t\,, \mathrm{d}\boldsymbol{\psi}_t\rangle + \langle \mathrm{d}\boldsymbol{\psi}_t\,, \mathrm{d}\boldsymbol{\psi}_t\rangle$$

and yields

$$\mathrm{d}\|\psi_t\|^2 = 2\,\mathrm{Re}\,\langle\,\psi_t, (\mathbb{K} - \mathrm{Re}\langle\psi_t\,,\mathbb{K}\psi_t\rangle)\,\psi_t\,\rangle\;\mathrm{d}t$$
$$+\sum_{a=1}^{\mathscr{A}}\mathrm{d}\nu_{a,t}\left\langle\psi_t\,,\left(\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\psi_t\|}-\mathbb{1}\right)+\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\psi_t\|}-\mathbb{1}\right)^\dagger+\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\psi_t\|}-\mathbb{1}\right)^\dagger\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\psi_t\|}-\mathbb{1}\right)\right)\psi_t\right\rangle$$

We observe that if $\|\psi_t\|^2 = 1$ the coefficient of $\mathrm{d}t$ vanishes. Furthermore, under the same hypothesis

$$\left\langle\psi_t\,,\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\psi_t\|}-\mathbb{1}\right)^\dagger\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\psi_t\|}-\mathbb{1}\right)\psi_t\right\rangle=$$
$$\left\langle\psi_t\,,\left(\frac{\mathbb{A}_a^\dagger\mathbb{A}_a}{\|\mathbb{A}_a\psi_t\|^2}-\frac{\mathbb{A}_a^\dagger}{\|\mathbb{A}_a\psi_t\|}-\frac{\mathbb{A}_a}{\|\mathbb{A}_a\psi_t\|}+\mathbb{1}\right)\psi_t\right\rangle=-2\,\mathrm{Re}\left\langle\psi_t\,,\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\psi_t\|}-\mathbb{1}\right)\psi_t\right\rangle$$

whence the claim readily follows □

2.3. Summary of main features

- Stochastic process satisfying a stochastic differential equation in Ito sense.
- The stochastic differential equation is **non-linear**.
- The stochastic differential equation is **non-local**: all components of the state vector and not just the neighboring-ones are needed to determine the new value of individual components of the state vector.
- Non-linearity and non-locality are necessary to guarantee **pathwise probability conservation**

$$\mathrm{d}\left(\psi_t^\dagger\psi_t\right)=0$$

3. LINEAR EQUATION

As the origin of non-linearity resides in the preservation of the norm of the states vector, we recover a linear equation by looking for an equivalent norm non-preserving evolution. To this goal we set

$$\psi_t=\frac{\phi_t}{\|\phi_t\|} \tag{6}$$

Upon inserting (6) and (4) into (5) we get into

$$\mathrm{d}\frac{\phi_t}{\|\phi_t\|}=-\mathrm{d}t\left(\mathbb{K}-\frac{\mathrm{Re}\,\langle\,\phi_t\,,\mathbb{K}\,\phi_t\,\rangle}{\|\phi_t\|^2}\right)\frac{\phi_t}{\|\phi_t\|}+\sum_{a=1}^{\mathscr{A}}\mathrm{d}\nu_{a,t}\left(\frac{\|\phi_t\|\,\mathbb{A}_a}{\|\mathbb{A}_a\,\phi_t\|}-\mathbb{1}\right)\frac{\phi_t}{\|\phi_t\|}$$

The rules of differentiation along stochastic paths give

$$\mathrm{d}\frac{\phi_t}{\|\phi_t\|}=\frac{(\mathrm{d}\phi_t)}{\|\phi_t\|}+\phi_t\mathrm{d}\frac{1}{\|\phi_t\|}+(\mathrm{d}\phi_t)\mathrm{d}\frac{1}{\|\phi_t\|}$$

We now surmise that the vector ϕ_t solves the linear stochastic differential equation

$$\mathrm{d}\phi_t=\mathrm{d}t\,\mathbb{G}\phi_t+\sum_{a=1}^{\mathscr{A}}\mathrm{d}\nu_{a,t}\mathbb{N}_a\phi_t \tag{7}$$

Then we find

$$\mathrm{d}\frac{1}{\|\phi_t\|}=-\frac{\langle\phi_t\,,\left(\mathbb{G}+\mathbb{G}^\dagger\right)\phi_t\rangle}{2\,\|\phi_t\|^3}\mathrm{d}t+\sum_{a=1}^{\mathscr{A}}\mathrm{d}\nu_{a,t}\left(\frac{1}{\|\phi_t+\mathbb{N}_a\phi_t\|}-\frac{1}{\|\phi\|_t}\right)$$

and

$$(\mathrm{d}\phi_t)\mathrm{d}\frac{1}{\|\phi_t\|}=\sum_{a=1}^{\mathcal{A}}\mathrm{d}\nu_{a,t}\mathbb{N}_a\phi_t\left(\frac{1}{\|\phi_t+\mathbb{N}_a\phi_t\|}-\frac{1}{\|\phi_t\|}\right)$$

Gathering all contributions we get

$$\mathrm{d}\frac{\phi_t}{\|\phi_t\|}=\mathrm{d}t\,\frac{\mathbb{G}\phi_t}{\|\phi_t\|}-\mathrm{d}t\,\phi_t\frac{\langle\phi_t\,,\left(\mathbb{G}+\mathbb{G}^{\dagger}\right)\phi_t\rangle}{2\|\phi_t\|^3}+\sum_{a=1}^{\mathcal{A}}\mathrm{d}\nu_{a,t}\left(\frac{\mathbb{N}_a\phi_t}{\|\phi_t\|}+(\mathbb{1}+\mathbb{N}_a)\phi_t\left(\frac{1}{\|\phi_t+\mathbb{N}_a\phi_t\|}-\frac{1}{\|\phi_t\|}\right)\right)$$

We thus derive that self-consistence of the linear surmise (7) requires

$$-\left(\mathbb{K}-\frac{\operatorname{Re}\langle\,\phi_t\,,\mathbb{K}\,\phi_t\,\rangle}{\|\phi_t\|^2}\right)\phi_t=\mathbb{G}\phi_t-\phi_t\frac{\langle\phi_t\,,\left(\mathbb{G}+\mathbb{G}^{\dagger}\right)\phi_t\rangle}{2\,\|\phi_t\|^2}$$

and

$$\left(\frac{\|\phi_t\|\,\mathbb{A}_a}{\|\mathbb{A}_a\,\phi_t\|}-\mathbb{1}\right)\frac{\phi_t}{\|\phi_t\|}=\frac{\mathbb{N}_a\phi_t}{\|\phi_t+\mathbb{N}_a\phi_t\|}+\phi_t\left(\frac{1}{\|\phi_t+\mathbb{N}_a\phi_t\|}-\frac{1}{\|\phi\|_t}\right)$$

to hold true. We satisfy such conditions if we set

$$\mathbb{G}=-\mathbb{K}$$

and, for all $a=1,\ldots,\mathcal{A}$

$$\mathbb{N}_a=\mathbb{A}_a-\mathbb{1}$$

We have thus proved that

Proposition. *A vector ϕ_t related to the state vector*

$$\psi_t=\frac{\phi_t}{\|\phi_t\|}$$

solution of (1) obeys the linear stochastic differential equation with Poisson noise

$$\mathrm{d}\phi_t=-\mathrm{d}t\,\mathbb{K}\phi_t+\sum_{a=1}^{\mathcal{A}}\mathrm{d}\nu_{a,t}\left(\mathbb{A}_a-\mathbb{1}\right)\phi_t \tag{8}$$

3.1. Reduction to a pure jump process

We may apply a change of variables, "the removal of the drift" to couch the piecewise deterministic equation (8) into an equation for a **pure jump** process with time dependent rates.

$$\phi_t=\mathbb{F}_{tt_\iota}\varphi_t$$

with

$$\begin{aligned}\dot{\mathbb{F}}_{tt_\iota}&=-\mathrm{d}t\,\mathbb{K}\,\mathbb{F}_{tt_\iota}\\ \mathbb{F}_{t_\iota t_\iota}&=\mathbb{1}\end{aligned}$$

The equation for the jump process is

$$\mathrm{d}\varphi_t=\sum_{a=1}^{\mathcal{A}}\mathrm{d}\nu_{t\,a}\left(\mathbb{F}_{tt_\iota}^{-1}\mathbb{A}_a\mathbb{F}_{tt_\iota}-\mathbb{I}\right)\varphi_t \tag{9}$$

with

$$\begin{aligned}\mathrm{d}\nu_{a,t}\mathrm{d}\nu_{a',t}&=\delta_{aa'}\mathrm{d}\nu_{a',t}\\ \mathrm{E}\left(\mathrm{d}\nu_{ta}|\varphi_t\right)&=\frac{\|\mathbb{A}_a\mathbb{F}_{tt_\iota}\varphi_t\|^2}{\|\mathbb{F}_{tt_\iota}\varphi_t\|^2}\,\mathrm{d}t\end{aligned}$$

We recover the stochastic state vector from the jump process from the identity

$$\psi_t=\frac{\mathbb{F}_{tt_\iota}\varphi_t}{\|\mathbb{F}_{tt_\iota}\varphi_t\|}$$

4. INTERPRETATION OF THE JUMP RATE

Before the first jump occurs, (8) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\phi_t\|^2 = -\sum_{a=1}^{\mathscr{A}} \|\mathbb{A}_a\, \phi_t\|^2$$

whence

$$\frac{\mathrm{d}}{\mathrm{d}t}\ln\|\boldsymbol{\phi}_t\|^2 = -\sum_{a=1}^{\mathscr{A}} \|\mathbb{A}_a \boldsymbol{\psi}_t\|^2 \equiv -\Gamma_t \leq 0$$

The physical interpretation of Γ is that of probability of observing a jump per unit of time. In other words, if we define the waiting time τ between two jumps then the waiting time cumulating function is

$$F_t = \mathrm{P}(\tau\,\leq\, t)$$

the relation between the waiting time probability density and the state vector is then

$$\mathrm{d}F_t = \mathrm{d}t\,\Gamma_t \exp\left(-\int_0^t \mathrm{d}s\,\Gamma_s\right) = -\mathrm{d}t\,\frac{\mathrm{d}}{\mathrm{d}t}\|\boldsymbol{\phi}_t\|^2$$

5. RECOVERY OF THE LINDBLAD–GORINI–KOSSAKOWSKI–SUDARSHAN EQUATION

Finally, we derive the Lindblad–Gorini–Kossakowski–Sudarshan master equation corresponding to the stochastic Schrödinger equation (1). The state operator solution of the Lindblad–Gorini–Kossakowski–Sudarshan master equation is the average over the realizations of the Poisson noise of the projector onto the stochastic state vector ψ

$$\boldsymbol{\rho}_t = \mathrm{E}\,\boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger$$

Upon differentiating with respect to the time variable t the afore identity, we get into

$$\begin{aligned}
\mathrm{d}\boldsymbol{\rho}_t &= \mathrm{d}\,\mathrm{E}\,\boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger = \mathrm{E}\left((\mathrm{d}\boldsymbol{\psi}_t)\,\boldsymbol{\psi}_t^\dagger + \boldsymbol{\psi}_t\mathrm{d}\boldsymbol{\psi}_t^\dagger + (\mathrm{d}\boldsymbol{\psi}_t)\,\mathrm{d}\boldsymbol{\psi}_t^\dagger\right) \\
&= -\,\mathrm{E}\left(\mathrm{d}t\,\big(\mathbb{K} - \mathrm{Re}\langle\boldsymbol{\psi}_t\,,\mathbb{K}\boldsymbol{\psi}_t\rangle\big)\boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger + \sum_{a=1}^{\mathscr{A}}\mathrm{d}\nu_{a,t}\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\boldsymbol{\psi}_t\|} - \mathbb{1}\right)\boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger\right) \\
&\quad + \mathrm{E}\left(\mathrm{d}t\,\boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger\big(\mathbb{K} - \mathrm{Re}\langle\boldsymbol{\psi}_t\,,\mathbb{K}\boldsymbol{\psi}_t\rangle\big) + \boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger\sum_{a=1}^{\mathscr{A}}\mathrm{d}\nu_{a,t}\left(\frac{\mathbb{A}_a^\dagger}{\|\mathbb{A}_a\,\boldsymbol{\psi}_t\|} - \mathbb{1}\right)\right) \\
&\quad + \sum_{a=1}^{\mathscr{A}}\mathrm{E}\,\mathrm{d}\nu_{a,t}\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\boldsymbol{\psi}_t\|} - \mathbb{1}\right)\boldsymbol{\psi}_t\,\boldsymbol{\psi}_t^\dagger\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\boldsymbol{\psi}_t\|} - \mathbb{1}\right)^\dagger
\end{aligned}$$

As all stochastic increments are evaluated in the **pre-point** discretization, we can apply the law of the iterated expectation which combined with (4) yields

$$\begin{aligned}
\mathrm{d}\boldsymbol{\rho}_t &= -\,\mathrm{E}\left(\mathrm{d}t\left(\mathbb{K} - \sum_{a=1}^{\mathscr{A}}\frac{\|\mathbb{A}_a\boldsymbol{\psi}_t\|^2}{2}\right)\boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger + \sum_{a=1}^{\mathscr{A}}\|\mathbb{A}_a\boldsymbol{\psi}_t\|^2\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\,\boldsymbol{\psi}_t\|} - \mathbb{1}\right)\boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger\right) \\
&\quad + \mathrm{E}\left(\mathrm{d}t\,\boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger\left(\mathbb{K} - \sum_{a=1}^{\mathscr{A}}\frac{\|\mathbb{A}_a\boldsymbol{\psi}_t\|^2}{2}\right) + \boldsymbol{\psi}_t\boldsymbol{\psi}_t^\dagger\sum_{a=1}^{\mathscr{A}}\|\mathbb{A}_a\boldsymbol{\psi}_t\|^2\left(\frac{\mathbb{A}_a^\dagger}{\|\mathbb{A}_a\,\boldsymbol{\psi}_t\|} - \mathbb{1}\right)\right) \\
&\quad + \sum_{a=1}^{\mathscr{A}}\mathrm{E}\,\|\mathbb{A}_a\boldsymbol{\psi}_t\|^2\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\boldsymbol{\psi}_t\|} - \mathbb{1}\right)\boldsymbol{\psi}_t\,\boldsymbol{\psi}_t^\dagger\left(\frac{\mathbb{A}_a}{\|\mathbb{A}_a\boldsymbol{\psi}_t\|} - \mathbb{1}\right)^\dagger
\end{aligned}$$

The result of cancellations between non-linear terms yields

$$\mathrm{d}\boldsymbol{\rho}_t = -\mathrm{d}t\,(\mathbb{K}\boldsymbol{\rho}_t + \boldsymbol{\rho}_t\,\mathbb{K}^\dagger) + \mathrm{d}t\sum_{a=1}^{\mathscr{A}}\mathbb{A}_a\,\boldsymbol{\rho}_t\,\mathbb{A}_a^\dagger$$

6. NUMERICAL ADVANTAGE OF THE STOCHASTIC SCHRÖDINGER EQUATION

In [4] are discussed three main reasons to introduce the stochastic Schrödinger equation.

- Indirect (continuous time) measurement: unraveling relates the statistics of individual random detection events to the state operator. Application example: quantum state parameter prediction and retrodiction.

- Numerical integration in high dimensional Hilbert spaces:

N-state system	Real numbers to store per step	Scaling of expected computing time
Direct integration of the master equation	$O(N^2)$	$O(N^4)$
Integration via unraveling	$O(2\,N)$	$O(\mathcal{N} \times N^2)$ $\mathcal{N} = \#$ (realizations)

- Foundational reason: element of a still missing theory of quantum state reduction? [2]

* * *

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