Random conformally invariant curves

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Foreword.

This minicourse is about conformally invariant random curves in two dimensional domains. The study of these curves is motivated by critical phenomena in statistical physics: it can be generally argued that models of statistical mechanics at their critical points of continuous phase transitions should have scaling limits which exhibit conformal symmetry. The random curves we consider are the natural candidates for scaling limits of interfaces arising in these models — some such interfaces are illustrated in Figures 1, 2, 3. We will encounter different random curves, which are all described and constructed in a rather similar manner. They have become known collectively as Schramm-Loewner evolutions, stochastic Loewner evolutions, or briefly SLEs.

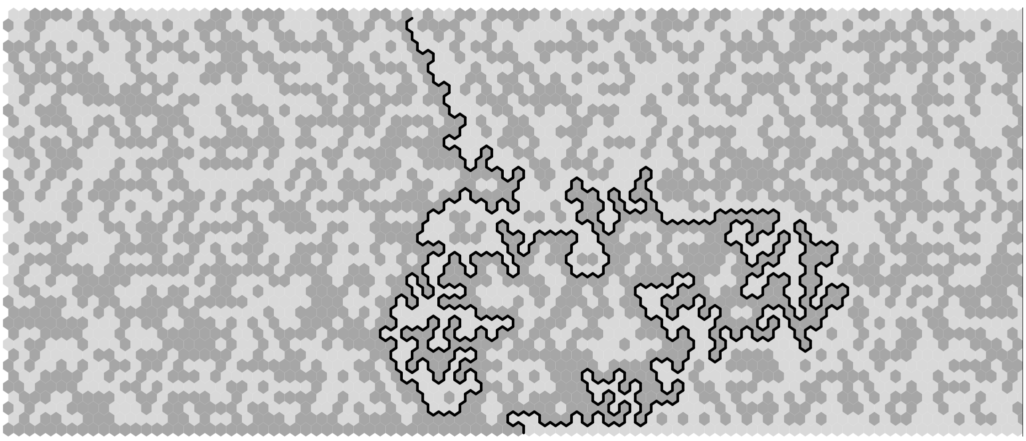

Figure 1: Exploration path of critical percolation on hexagonal lattice separates hexagons of the two different colours.

In this short time it is impossible to cover even the crucial parts of the theory in detail, so the aim is rather to introduce the SLE curves and give examples of some of the most common techniques that are needed when working with them. I regret that there will be no time to cover any of the topics in statistical mechanics, which are the real motivation for studying SLEs, and which feature remarkable recent achievements as well as important open research problems. For the reader who is interested in obtaining a more profound understanding of SLEs, there are review articles and overviews [KN04, BB06, Car05], lecture notes [Wer02, Bef10, Law10], a textbook [Law05], and finally of course research articles which would be too numerous to list here. From the ICM talks [Sch06, Smi06, Smi10] one gets a fair picture of the state of the current research.

References

Lectures

[Bef10] V. Beffara: "Schramm-Loewner evolutions and other conformally invariant objects". Clay Mathematics Summer School, Buzios, Brazil (2010).

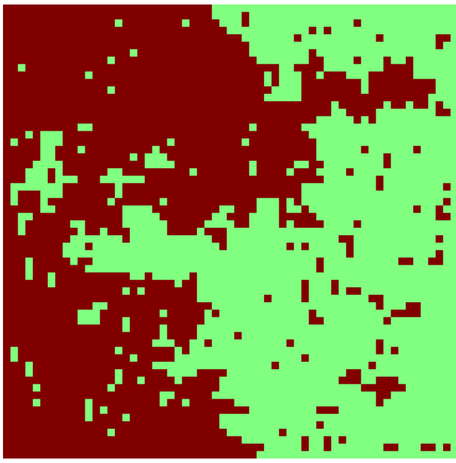

Figure 2: An interface separates different spin clusters in the critical Ising model with Dobrushin boundary conditions (simulation and picture by Antti Kemppainen).

Figure 3: Loop erasure of a random walk.

[Law10] G. Lawler: "Fractal and multifractal properties of SLE". Clay Mathematics Summer School, Buzios, Brazil (2010).

[Wer02] W. Werner: "Random planar curves and Schramm-Loewner evolutions" Saint-Flour summer school (2002).

ICM talks

[Sch06] O. Schramm: "Conformally invariant scaling limits (an overview and a collection of problems)" ICM 2006.

[Smi06] S. Smirnov: "Towards conformal invariance of 2D lattice models" ICM 2006.

[Smi10] S. Smirnov: "Discrete Complex Analysis and Probability" ICM 2010.

Reviews

[Car05] J. Cardy: "SLE for theoretical physicists". Ann. Physics, 318, 81-118 (2005).

[KN04] W. Kager and B. Nienhuis: "A guide to stochastic Loewner evolution and applications". J. Stat. Phys., 115, 1149-1229 (2004).

[BB06] M. Bauer and D. Bernard: "2D growth processes: SLE and Loewner chains". Phys. Rep., 432, 115-222 (2006).

Textbook

[Law05] G. Lawler: "Conformally invariant processes in the plane". American Mathematical Society, 114 (2005).

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