Constructing interacting QFTs via OPEs

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- Two of these three parameters can be lifted (e.g., perturbation theory, YM theory with IR cutoff in 4D, ϕ^4_4 with UV cutoff)

Correlation functions and the OPE

What do we want

Correlation functions and the OPE

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- Coefficients have scaling degree $\sum_i[A_i]-[B]$ (homogeneous distributions up to logarithmic corrections)
- Sum is asymptotic: including all operators up to a fixed dimension $[B]\leq d$ in the sum, the difference between left- and right-hand side vanishes if all x_i scale together

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- Convergent instead of asymptotic sum

The flow equation framework

What has been shown

The flow equation framework

- Use L^{Λ,Λ_0}, the generating functional of connected, amputated correlation functions with insertions of $\mathcal{O}_{A_i}$:
  $L^{\Lambda,\Lambda_0}(\{\mathcal{O}_{A_i}\};\Phi) = \sum_k \Phi^k \langle 0|\mathcal{O}_{A_1}\cdots\mathcal{O}_{A_n}\phi_1\cdots\phi_k|0\rangle_{\mathrm{c,a}}$

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  $-\hbar\ln\int\exp\left(-\frac{1}{\hbar}\phi C^{-1}_{\Lambda,\Lambda_0}\phi - \frac{1}{\hbar}L_{\mathrm{int}}(\phi+\Phi)\right)\prod_{i=1}^n \mathcal{O}_{A_i}(\phi+\Phi,x_i)\mathcal{D}\phi$

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- $C^{\Lambda,\Lambda_0} = (k^2+m^2)^{-1}\left[\mathrm{e}^{-\frac{k^2+m^2}{\Lambda_0^2}} - \mathrm{e}^{-\frac{k^2+m^2}{\Lambda^2}}\right]$ is a regularised propagator with infrared cutoff Λ and ultraviolet cutoff Λ_0

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- No Feynman diagrams, no forest formula

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- $$\partial_\Lambda L^{\Lambda,\Lambda_0}(\mathcal{O}_A) = \frac{\hbar}{2}\frac{\delta}{\delta\Phi} \cdot \left(\partial_\Lambda C^{\Lambda,\Lambda_0}\right) \cdot \frac{\delta}{\delta\Phi} L^{\Lambda,\Lambda_0}(\mathcal{O}_A) - \left(\frac{\delta}{\delta\Phi} L^{\Lambda,\Lambda_0}\right) \cdot \left(\partial_\Lambda C^{\Lambda,\Lambda_0}\right) \cdot \left(\frac{\delta}{\delta\Phi} L^{\Lambda,\Lambda_0}(\mathcal{O}_A)\right), \text{ etc.}$$
- Boundary conditions: for $\Lambda = \Lambda_0$ we recover "bare" theory: $L^{\Lambda_0,\Lambda_0} = L_{\text{int}}$
- Formal expansion in $\hbar$ (perturbation theory, loop order ℓ) and number n of Φ fields gives $\mathcal{L}_n^{\Lambda,\Lambda_0,\ell}(\mathbf{k})$ in momentum space
- No Feynman diagrams, no forest formula
- Idea by Polchinski, mathematically rigorous by Keller/Kopper

Proving stuff

- Flow equation:

$$\partial_\Lambda \mathcal{L}_n^{\Lambda,\Lambda_0,\ell}(\mathbf{k}) = \frac{1}{2}\int \left(\partial_\Lambda C^{\Lambda,\Lambda_0}(p)\right) \mathcal{L}_{n+2}^{\Lambda,\Lambda_0,\ell-1}(\mathbf{k},p,-p)\frac{\mathrm{d}^4 p}{(2\pi)^4}$$
$$-\frac{1}{2}\sum_{n'=0}^{n}\sum_{\ell'=0}^{\ell} \mathcal{L}_{n'+1}^{\Lambda,\Lambda_0,\ell'}(\mathbf{k},-q)\left(\partial_\Lambda C^{\Lambda,\Lambda_0}(q)\right) \mathcal{L}_{n-n'+1}^{\Lambda,\Lambda_0,\ell-\ell'}(q,\mathbf{k})$$

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- Additional momentum derivatives ∂^w are needed to close induction ($\mathcal{L}_n^{\Lambda,\Lambda_0,\ell}$ smooth in $\mathbf{k}$ for all $\Lambda < \Lambda_0$)

Proving stuff

- Bounds: $\left\|\partial^w \mathcal{L}_n^{\Lambda,\Lambda_0,\ell}(\mathbf{k})\right\| \leq \sup(\Lambda, m)^{4-n-|w|}\mathcal{P}\left(\ln_+ \frac{\Lambda}{m}, \ln_+ \frac{|\mathbf{k}|}{m}\right)$
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$\leq (N!)^{-\frac{1}{2}} \sup(m,\Lambda)^{n+N} K^N |x|^{N-[\mathcal{O}_A]-[\mathcal{O}_B]} \|\Phi\|_{(N+2)(\ell+5)/2}$
- OPE is convergent for arbitrary (spacelike) separations!

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- UV- and IR-finite integral (subtraction of problematic terms due to factorisation condition), BPHZ-like renormalisation conditions

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Gauge theories

What I have done

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- Define antibracket $(A, B) \equiv \frac{\delta A}{\delta \phi} \cdot \frac{\delta B}{\delta \phi^\ddagger} - (\leftrightarrow)$ and action $S = S_{\mathrm{cl}} + S_{\mathrm{gf}} + S_{\mathrm{gh}} + S_{\mathrm{af}}$ with $(S, S) = 0$ (BRST invariance)

Gauge theories

- Instead of ϕ, we have gauge potential A^a_μ
- Action $S_{\text{cl}} = \frac{1}{4}\int F^{a\mu\nu}F^a_{\mu\nu}\,\mathrm{d}^4x$ with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \mathrm{i}gf^{abc}A^b_\mu A^c_\nu$ is invariant under gauge transformation $A^a_\mu \to A^a_\mu + \partial_\mu f^a + \mathrm{i}gf^{abc}A^b_\mu f^c$ for any f^c
- Problem: impossible to define a propagator
- Solution: Batalin-Vilkovisky (field-antifield) construction = extension of BRST = extension of Fadeev-Popov
- Enlarge field space (ghost c^a, antighost $\bar{c}^a$, auxiliary field B^a), antifields for each field $(A^\ddagger_{a\mu}, B^\ddagger_a, c^\ddagger_a, \bar{c}^\ddagger_a)$
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- Define nilpotent Slavnov-Taylor differential $\hat{s}G \equiv (S, G)$ (gauge transformation replaced by $\hat{s}A^a_\mu = \partial_\mu c^a + \mathrm{i}gf^{abc}A^b_\mu c^c$)

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- Strategy: First bound massless theory for any field content, then (for gauge theories) restore BRST invariance and obtain proper Ward identities in the physical limit $\Lambda \to 0$, $\Lambda_0 \to \infty$

Gauge theories

- Bounds in general case: $\left\|\partial^w \mathcal{L}^{\Lambda,\Lambda_0,\ell}_{\mathbf{KL}^\ddagger}(\mathbf{k})\right\| \leq$
$$\sum_{T\in\mathcal{T}_{m+n}} \mathsf{G}^{T,w}_{\mathbf{KL}^\ddagger}(\mathbf{k};\mu,\Lambda)\,\mathcal{P}\left(\ln_+ \frac{\sup(\mu,|\mathbf{k}|)}{\sup(\inf(\mu,\eta(\mathbf{k})),\Lambda)}, \ln_+ \frac{\Lambda}{\mu}\right)$$

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- Bounds are finite for non-exceptional momenta $\eta(\mathbf{k}) > 0$ as expected (known IR divergence for $\eta(\mathbf{k}) = 0$)

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- Define regularised antibracket $(\cdot,\cdot)^{\Lambda_0}$ and $S^{\Lambda_0}(g=0)$ (with $\exp\left(-p^2/\Lambda_0^2\right)$ at appropriate places), and set $\hat{s}_0^{\Lambda_0} F = (S^{\Lambda_0}(g=0), F)^{\Lambda_0}$ (linear, nilpotent, finite as $\Lambda_0 \to \infty$)

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- Ward identity for OPE coefficients: $\sum_{k=1}^{s} \sum_{C:\,[\mathcal{O}_C]\le[\mathcal{O}_{A_k}]+1} \mathcal{Q}_{A_k}{}^{C}\mathcal{C}^{B}_{A_1\cdots A_{k-1}CA_{k+1}\cdots A_s}(\mathbf{x}) =$ $\sum_C \mathcal{Q}_C{}^{B}\mathcal{C}^{C}_{A_1\cdots A_s}(\mathbf{x})$ with $\hat{q}\mathcal{O}_A = \sum_{B:\,[\mathcal{O}_A]+1} \mathcal{Q}_A{}^{B}\mathcal{O}_B$ (in particular, in the OPE of $\hat{q}$-invariant operators only $\hat{q}$-invariant operators appear on the right-hand side)

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- OPE is (at least) asymptotic:
  $\lim_{\tau\to 0} \tau^{[\mathcal{O}_{\mathbf{A}}]-D+\delta}\Big[\langle\Psi|\mathcal{O}_{A_1}(\tau x_1)\cdots\mathcal{O}_{A_s}(\tau x_s)|\Psi\rangle -$
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- Factorisation:
  $\mathcal{C}^{C}_{A_1\cdots A_n}(x_1,\ldots,x_n) = \sum_B \mathcal{C}^{B}_{A_1\cdots A_k}(x_1,\ldots,x_k)\mathcal{C}^{C}_{BA_{k+1}\cdots A_n}(x_k,\ldots,x_n)$
  holds for all $\frac{\max_{1\le i\le k}|x_i-x_k|}{\min_{k<j\le n}|x_i-x_k|} < 1$

Recursive construction

- Coupling constant derivative:

$$\partial_g \mathcal{C}^{B}_{A_1\cdots A_s}(\mathbf{x}) = \int \sum_{E:\ 1\le[\mathcal{O}_E]\le 4} \mathcal{I}^{E}\Big[-\mathcal{C}^{B}_{EA_1\cdots A_s}(y,\mathbf{x}) + \sum_{C:\ [\mathcal{O}_C]<[\mathcal{O}_B]} \mathcal{C}^{C}_{A_1\cdots A_s}(\mathbf{x})\mathcal{C}^{B}_{EC}(y,x_s) + \sum_{k=1}^{s}\sum_{C:\ [\mathcal{O}_C]\le[\mathcal{O}_{A_k}]} \mathcal{C}^{C}_{EA_k}(y,x_k)\mathcal{C}^{B}_{A_1\cdots A_{k-1}CA_{k+1}\cdots A_s}(\mathbf{x})\Big]\, \mathrm{d}^4 y$$

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$$\partial_g \mathcal{C}_{A_1\cdots A_s}^{B}(\mathbf{x}) = \int \sum_{E:\ 1\leq[\mathcal{O}_E]\leq 4} \mathcal{I}^E \Big[-\mathcal{C}_{EA_1\cdots A_s}^{B}(y,\mathbf{x}) + \sum_{C:\ [\mathcal{O}_C]<[\mathcal{O}_B]} \mathcal{C}_{A_1\cdots A_s}^{C}(\mathbf{x})\mathcal{C}_{EC}^{B}(y,x_s) + \sum_{k=1}^{s}\sum_{C:\ [\mathcal{O}_C]\leq[\mathcal{O}_{A_k}]} \mathcal{C}_{EA_k}^{C}(y,x_k)\mathcal{C}_{A_1\cdots A_{k-1}CA_{k+1}\cdots A_s}^{B}(\mathbf{x})\Big]\, \mathrm{d}^4y$$
- Quantum BRST differential:
$$\partial_g \mathcal{Q}_A{}^B = \int \sum_{E:\ 1\leq[\mathcal{O}_E]\leq 4} \mathcal{I}^E \Big[\sum_{C:\ [\mathcal{O}_C]\leq[\mathcal{O}_A]} \mathcal{C}_{EA}^{C}(y,0)\mathcal{Q}_C{}^B - \sum_{C:\ [\mathcal{O}_B]\leq[\mathcal{O}_C]\leq[\mathcal{O}_A]+1} \mathcal{Q}_A{}^C\mathcal{C}_{EC}^{B}(y,0)\Big]\, \mathrm{d}^4y$$

Recursive construction

- Coupling constant derivative:
  $\partial_g \mathcal{C}^B_{A_1\cdots A_s}(\mathbf{x}) = \int \sum_{E:\, 1\leq[\mathcal{O}_E]\leq 4} \mathcal{I}^E \Big[-\mathcal{C}^B_{EA_1\cdots A_s}(y,\mathbf{x}) + \sum_{C:\, [\mathcal{O}_C]<[\mathcal{O}_B]} \mathcal{C}^C_{A_1\cdots A_s}(\mathbf{x}) \mathcal{C}^B_{EC}(y, x_s) + \sum_{k=1}^s \sum_{C:\, [\mathcal{O}_C]\leq[\mathcal{O}_{A_k}]} \mathcal{C}^C_{EA_k}(y, x_k) \mathcal{C}^B_{A_1\cdots A_{k-1} C A_{k+1}\cdots A_s}(\mathbf{x}) \Big] \, \mathrm{d}^4 y$
- Quantum BRST differential:
  $\partial_g \mathcal{Q}_A{}^B = \int \sum_{E:\, 1\leq[\mathcal{O}_E]\leq 4} \mathcal{I}^E \Big[\sum_{C:\, [\mathcal{O}_C]\leq[\mathcal{O}_A]} \mathcal{C}^C_{EA}(y,0) \mathcal{Q}_C{}^B - \sum_{C:\, [\mathcal{O}_B]\leq[\mathcal{O}_C]\leq[\mathcal{O}_A]+1} \mathcal{Q}_A{}^C \mathcal{C}^B_{EC}(y,0) \Big] \, \mathrm{d}^4 y$
- Interaction operator $\mathcal{O}_\mathcal{I} = \sum_{E:\, 1\leq[\mathcal{O}_E]\leq 4} \mathcal{I}^E \mathcal{O}_E$ with $\mathcal{O}_\mathcal{I} = \partial_g L|_{g=0} + \mathcal{O}(g) + \mathcal{O}(\hbar)$ and $\hat{q}\mathcal{O}_\mathcal{I} = \mathrm{d}\mathcal{O}'$ for some $\mathcal{O}'$

Conclusion

What still needs to be done

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- Millenium prize problem: Prove confinement and the existence of a mass gap for gauge theories

Thank you for your attention

Questions?

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