

Limit shape for the $so(2n + 1)$ Lie algebras in the infinite rank limit and an electrostatic problem.

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Abstract

We consider a tensor power of the spinor representation of the Lie algebra $so(2n + 1)$. Tensor product decomposition into irreducible representations leads to the appearance of probability measure on the set of dominant integral weights. We consider the behavior of this measure in the limit of infinite tensor power and infinite rank of the algebra.

We show that in this limit the measure is concentrated in the single weight, thus the limit shape phenomenon is observed. The coordinates of this weight can be seen as the positions of charged particles on the line. These particles repulse each other but are confined in a closed interval by an external potential. We show that the limit shape is described by the density function of these particles.

Charge density function is the solution of a variational problem for the equilibrium measure, thus our result is comparable to various results in theory of random matrices and orthogonal polynomials.

Our result is similar to famous Vershik-Kerov-Logan-Shepp limit shape for the Plancherel measure on Young diagrams, since the limit shape for Young diagrams can be obtained from tensor product decomposition of $sl(n)$ -representations.