Demographic fluctuations in a population of anomalously diffusing individuals

Piero Olla

olla@dsf.unica.it

ISAC-CNR Sez. Cagliari

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- Extinction in time $t_{ext} \sim N_0 \Gamma^{-1}$.

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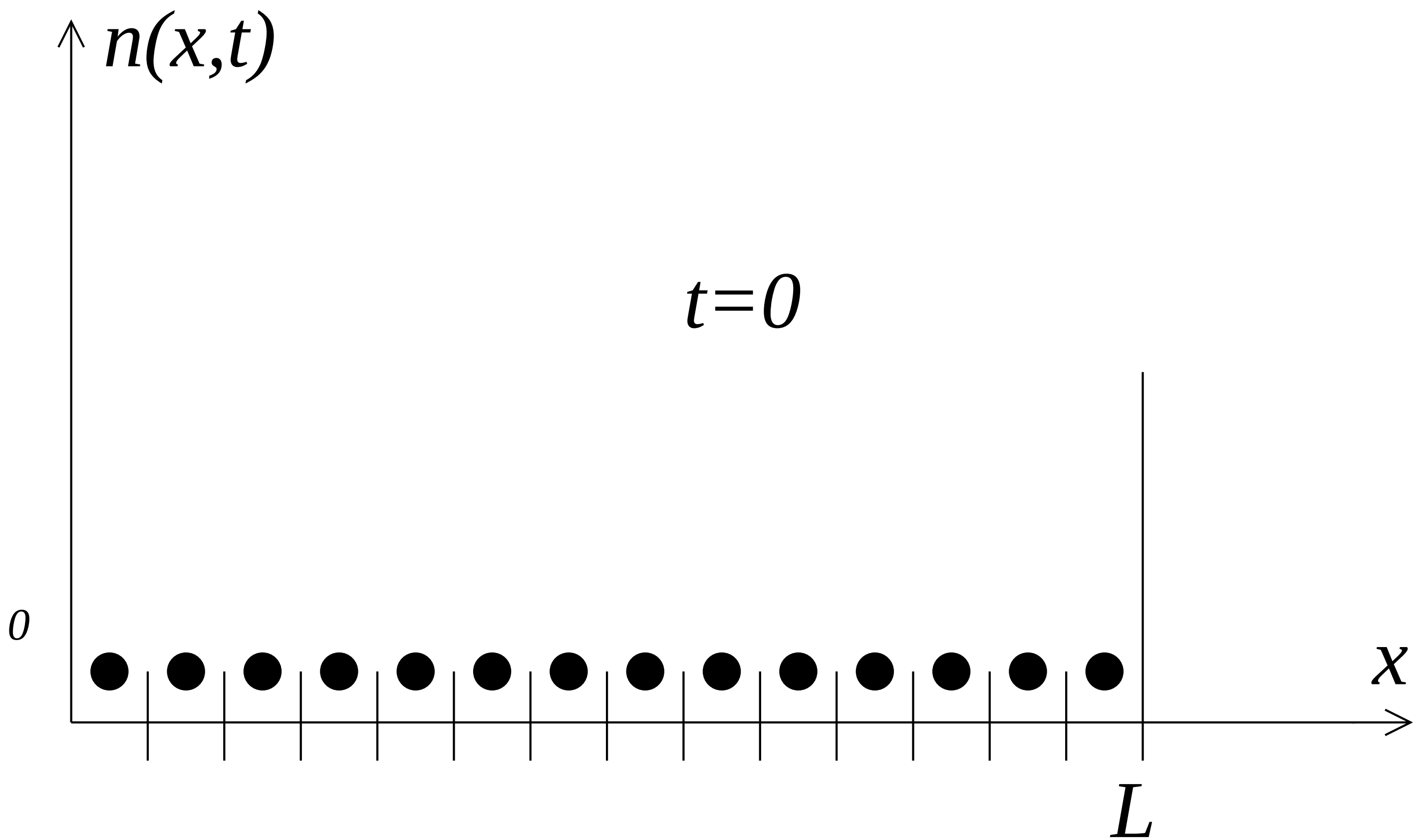
- Summing up to time t:

$$\sigma^2_{N(t)} = 2N_0 \Gamma t$$

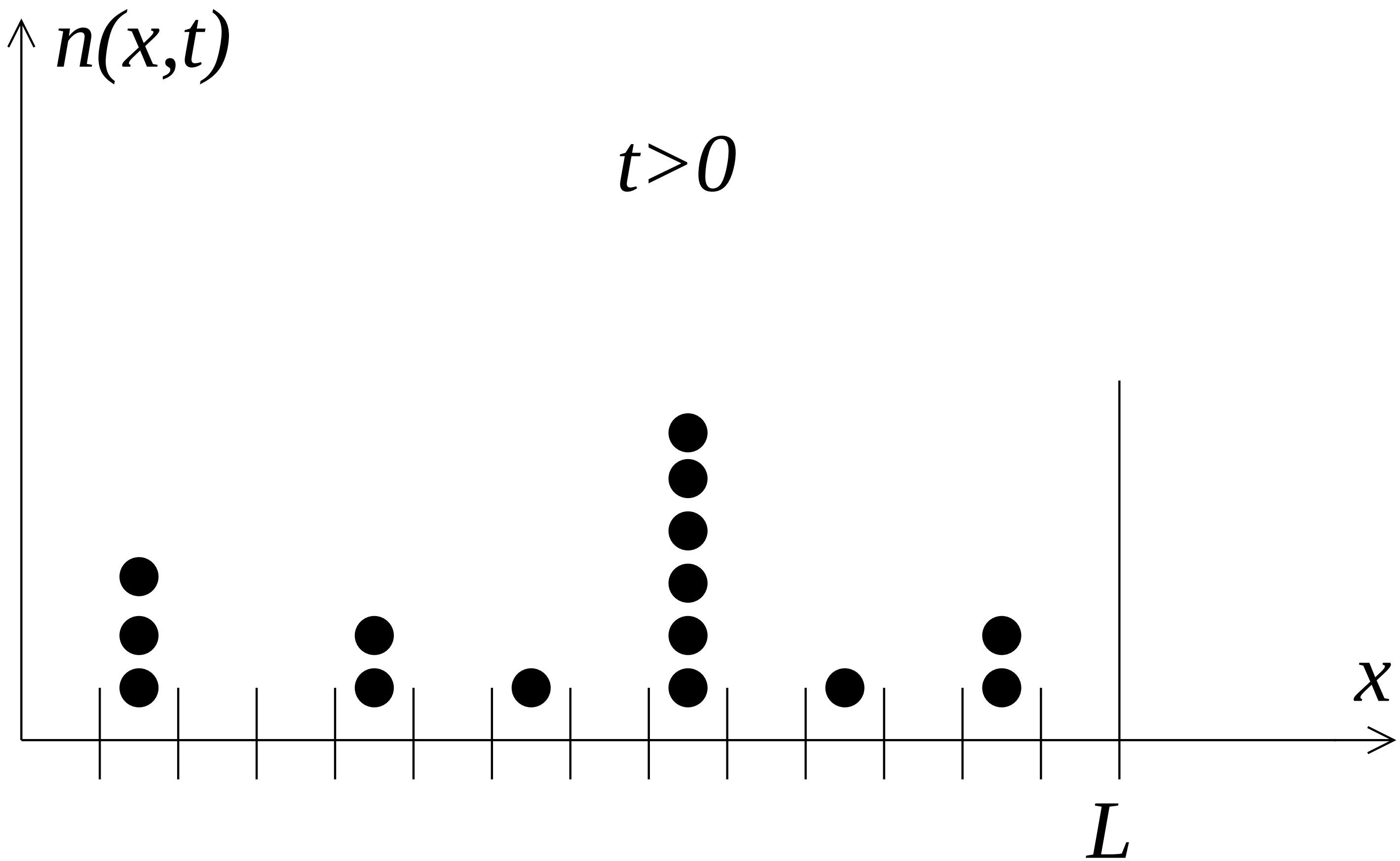
Including some spatial structure

Divide the domain in cells and distribute bugs uniformly in them



Wait some time...

Including some spatial structure



Taller towers balance on the average increasingly wider gaps where the bugs have gone extinct.

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  - Constant mean density if bugs are initially uniformly distributed:

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  - Clustering behavior in $D \leq 2$:

$$\sigma^2_{n(x,t)} \propto t^{1/2} \quad (D=1); \quad \sigma^2_{n(x,t)} \propto \ln t \quad (D=2).$$

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- Correlation length of fluctuations determined by Brownian motion scale $\lambda(t) \sim (\kappa t)^{1/2}$.

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- $\sigma^2 \propto t$ maximal growth in Galton-Watson case.
- Recall $\lambda(t) \propto t^{1/2}$, that precludes clustering for $D = 3$.

We wander whether we could extend the approach to more general (anomalous) diffusion processes.

Practical relevance

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- Migration in the presence of long jumps (Lévy flights $\Rightarrow$ superdiffusion).
- Ageing in mutation phenomena (migration in genotype space; selection as a form of clustering in genotype space).

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- Four classes could roughly be identified
  - Gaussian processes, such as the fractional Brownian motion; individuals that migrate with a power-law correlated velocity (generalized Langevin equation).
  - Continuous time random walk (CTRW): discrete jumps are separated by waiting time characterized by a distribution with heavy tails.
  - Migration in a spatial assembly of random traps
  - Migration by Lévy flights (not properly a diffusion process).

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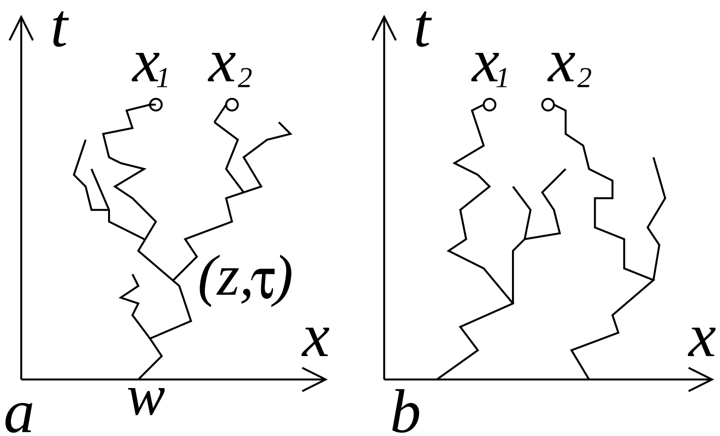

- Fluctuations are accounted for by the connected family trees in (a).
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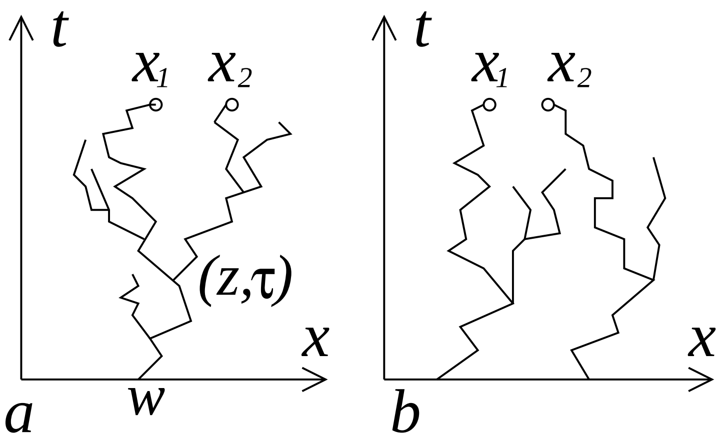

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- It is not clear whether the bug generated in (z, τ) should preserve memory of the trajectory followed by its ancestors or not.

A closer look at connected trees

$$\begin{aligned}\rho_{2c}(x_1, x_2; t) &= 2\Gamma \int_0^t \mathrm{d}\tau \int \mathrm{d}z \int \mathrm{d}w \; \rho_1(x_1, t|z, \tau; w, 0) \\ &\times \rho_1(x_2, t|z, \tau; w, 0)\rho_1(z, \tau|w, 0)\rho_1(w, 0).\end{aligned}$$

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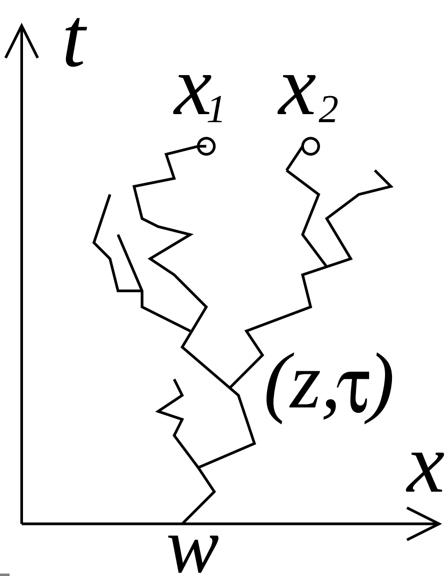

- As already stated, to calculate the connected correlation $\rho_{2c}(x_1, x_2; t)$, we must consider family trees like the one beside.
- As diffusion is non-Markovian, the conditioning on $(w, 0)$ in the $\rho_1(x_{1,2}, t|z, \tau; w, 0)$ entering the equation for ρ_{2c} is not automatically irrelevant.

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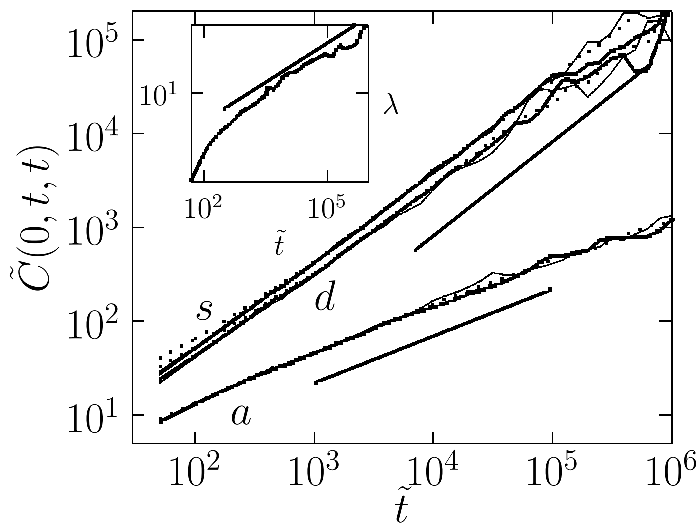

- Growth of σ_n^2 for $D = 1$ and $H = 0.25$ (subdiffusion). Case (a) is the one with no memory. Insert: scaling of the correlation length $\lambda(t)$. The slopes $t^{1/2}$ are shown for comparison.

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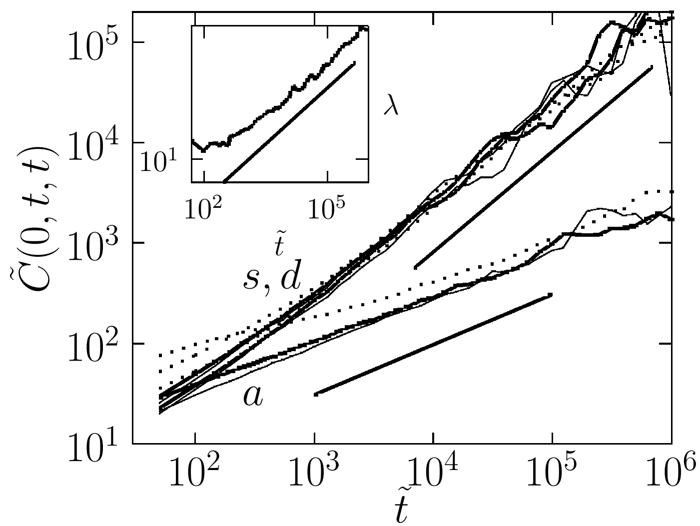
- Again, in the case of a CTRW:



- Growth of σ_n^2 for $D = 1$ and $H = 0.75$ (superdiffusion). The steeper lines come from including memory in a CTRW (s) and working with traps (d) (tough trouble; wait and see).

Memory + Gaussian diffusion

- Recall the equation for ρ_{2c}:

$$\begin{aligned}\rho_{2c}(x_1, x_2; t) &= 2\Gamma \int_0^t \mathrm{d}\tau \int \mathrm{d}z \int \mathrm{d}w \; \rho_1(x_1, t | z, \tau; w, 0) \\ &\times \rho_1(x_2, t | z, \tau; w, 0) \rho_1(z, \tau | w, 0) \rho_1(w, 0).\end{aligned}$$

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- Carrying out the integrals we get after simple algebra

$$\begin{aligned}\rho_{2c}(x_1,x_2;t) &= \frac{\Gamma n_0}{2\sqrt{\pi}}\int_0^t \frac{\mathrm{d}\tau}{\sigma(t;\tau)}\exp\Big[-\frac{(x_1-x_2)^2}{4\sigma^2(t,\tau)}\Big].\\ \sigma^2(t,\tau) &= \sigma^2(t)-\frac{\langle y(t)y(\tau)\rangle^2}{\sigma^2(\tau)};\\ \sigma^2(t) &= \kappa_H|t|^{2H};\qquad y(t)=x(t)-x(0).\end{aligned}$$

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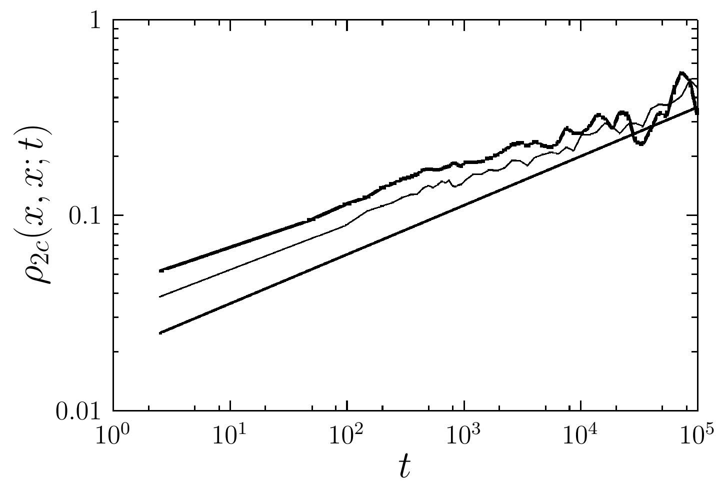

- Fluctuation build-up for a Gaussian superdiffusive process ($H = 0.75$; heavy line). The line t^{1-H} is shown for comparison. Thin line obtained from Lévy flights (wait and see).

The other kingdom

- Non-Gaussian processes
  - CTRW-bugs: two throws of dice at each time: one to decide how long to wait; one to decide where to go. Waiting-time PDF with heavy tail to produce anomalous diffusion. Subdiffusion easy to get.
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- Notice that Lévy bugs are Markovian; hence, they do not have a prescription problem.
- Anomalous scaling is produced in the two cases by events in the heavy tails. Migration by CTRW is dominated by the longer waiting times; migration by Lévy flights is dominated by the longer jumps (only superdiffusion possible in this case).

Lévy flights

- Markovian dynamics $\Rightarrow$ evolution equation for ρ_{2c} local in time

$$\rho_{2c}(x_1, x_2; t + \Delta t) = \int \mathrm{d}y_1 \int \mathrm{d}y_2 \; \rho_1(x_1, t + \Delta t | y_1, t)$$
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- Power-law tails: $\rho_1(x, \Delta t|0, 0) \propto |x|^{-1-\beta}, \; 0 < \beta < 1$ $\Rightarrow \rho_{1k}(\Delta t) \simeq 1 - \alpha|k|^\beta \Delta t$. Notice $\alpha^{2/\beta} \sim (\Delta x)^2/(\Delta t)^{1/\beta}$, diffusivity-like quantity.

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Anomalous “diffusion” again

- The Lévy flights are characterized by infinite second moment $\langle |x(t) - x(0)|^2 \rangle = \infty$ also for a single jump. Nevertheless, $\rho_{1k}(\Delta t) \simeq 1 - \alpha |k|^\beta \Delta t$ identifies a characteristic length $y(t) \propto t^{1/\beta}$ that gives the scale of the clusters at time t.

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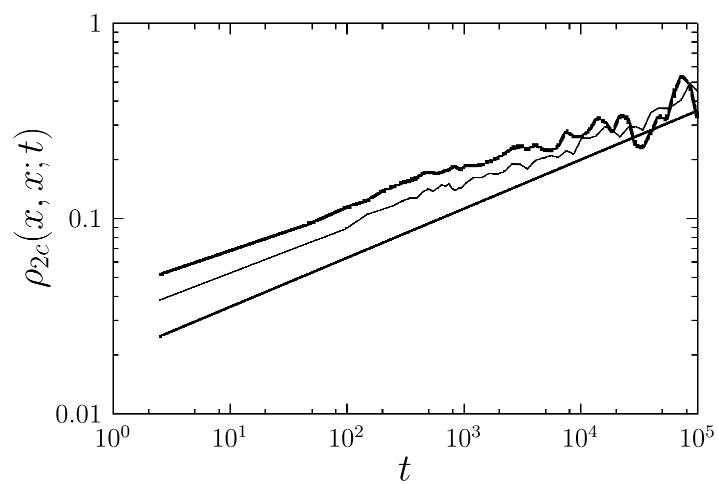
Recall the behavior ($1/\beta = 0.75$; thin line; the heavy line described Gaussian superdiffusion).



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- This is the same behavior of bugs that do not migrate!

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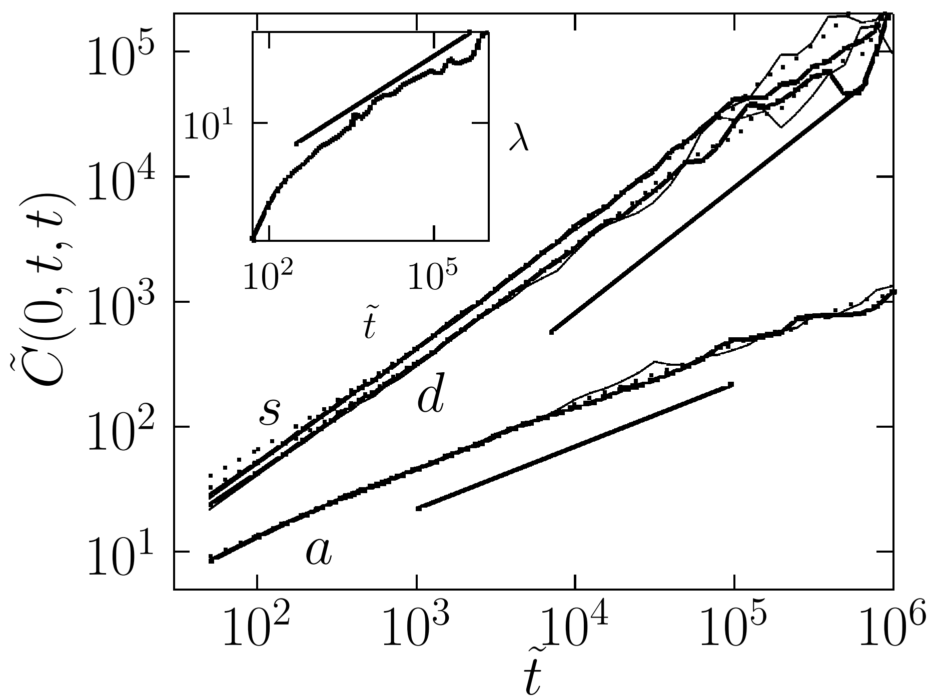
- However, “simply” social bugs are already characterized by a maximal level of fluctuation ($\sigma_n^2 \propto t$) and random traps make bugs to disperse even less than in the simple social case.
- We expect the fluctuation growth in the case of random traps to remain maximal.

Numerical results



- Growth of $C(0;t,t)$ in $D=1$ and subdiffusive regime. Antisocial bugs (a), social bugs (s) and bugs in random traps (d). Insert: scaling of $\lambda(t)$ in the “antisocial case”.

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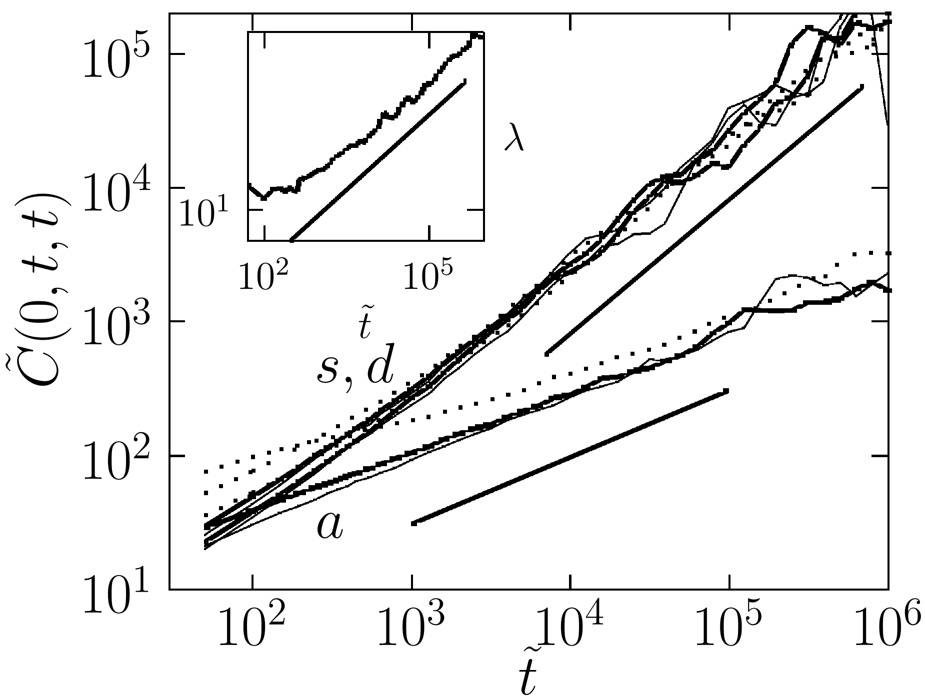

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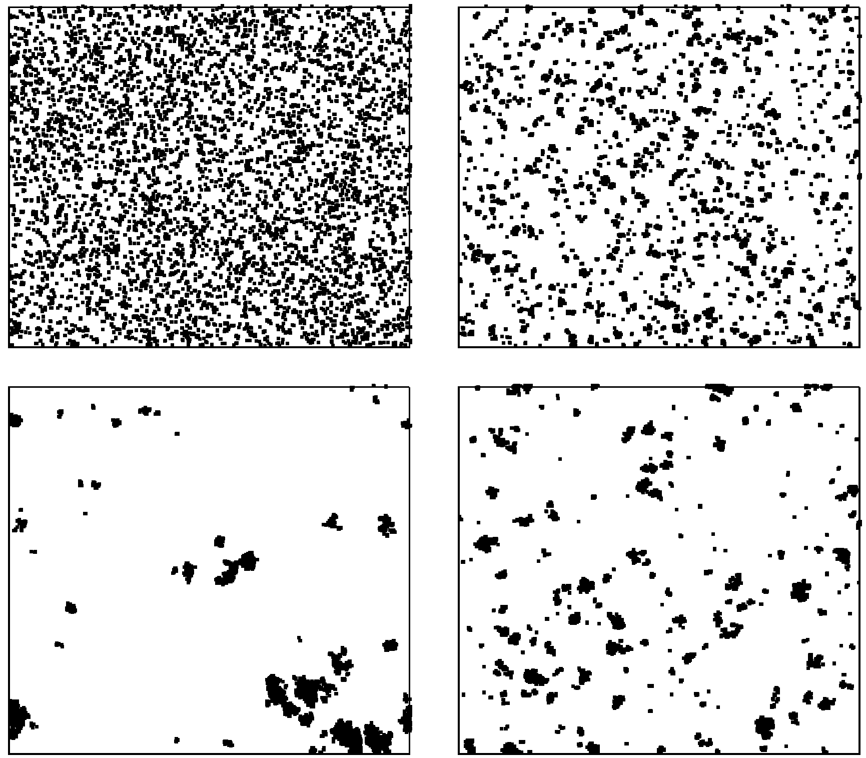

- Sequence of snapshots of a population in a 2D field of traps. A small diffusivity is added to mimick the effect of small scale individual motion.

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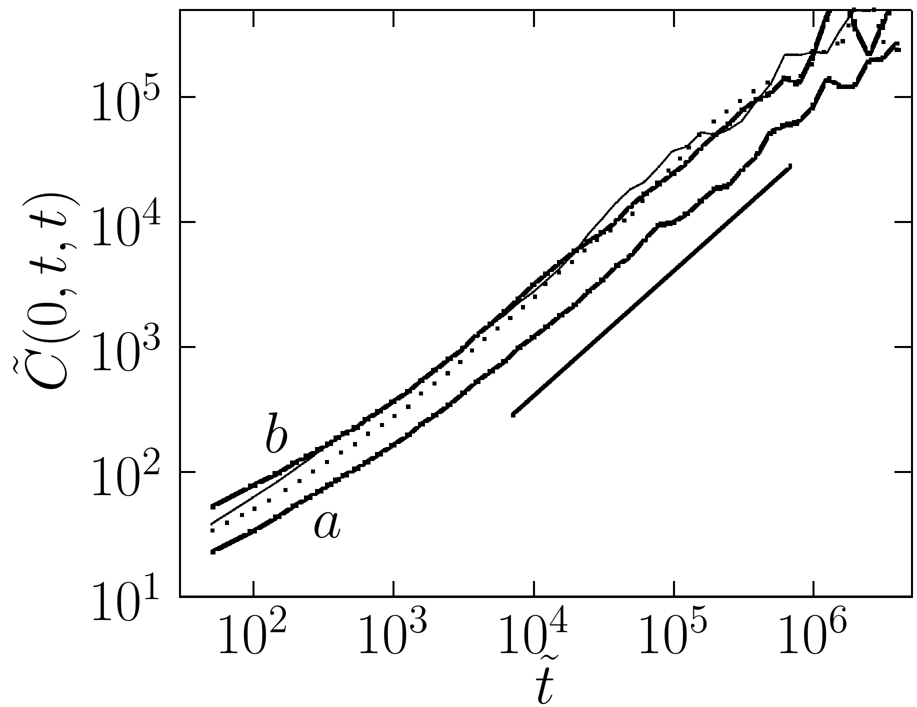

- Growth of $C(0;t,t)$ in $D=2$ (a) and $D=1$ (b) for different mean numbers of bugs per trap (values range from 1 to 200).

Summarizing

- Gaussian anomalous diffusion (FBM, individuals moving with velocity that is solution of a GLE, and others): if offsprings share memory of the trajectories with their parents, demographic fluctuations will scale anomalously. Otherwise, the case of Brownian bugs is recovered.

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- A spatial assembly of random traps: the dynamics falls back again on that of a Galton-Watson process.

Conclusion

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- Refs.: P. Olla, Phys. Rev E (2011); P. Olla, arXiv:1208.5868