THE DE BROGLIE-BOHM THEORY-NON-LOCALITY AND ANSWERS TO OBJECTIONS.

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SUMMARY

The de Broglie-Bohm theory is a theory of particles in motion that are guided by the quantum state. If we assume quantum equilibrium ($\rho = |\Psi|^2$) at some initial time, we get it for all times.

Therefore one recovers the usual quantum predictions for particle positions.

But, through an analysis of “measurements”, for example those of “spin”, one also recovers the usual quantum predictions for all “observables”.

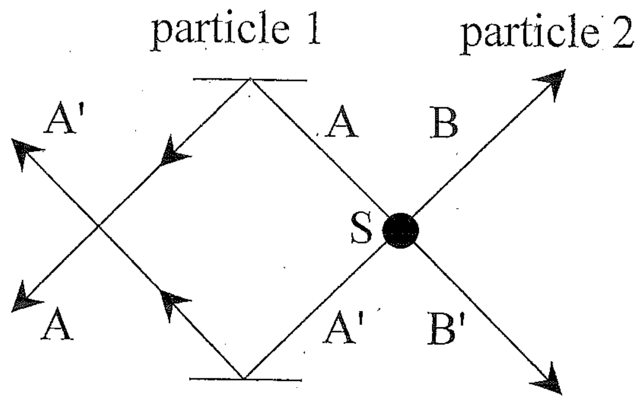
WHAT ABOUT NON-LOCALITY?

To illustrate the non-locality of de Broglie–Bohm theory, consider two particles in one dimension (to simplify matters), with (X_1, X_2) denoting the positions of the particles.

Write $\Psi(x_1, x_2, t) = R(x_1, x_2, t)e^{iS(x_1,x_2,t)}$, the guiding equation for particle 1 is:

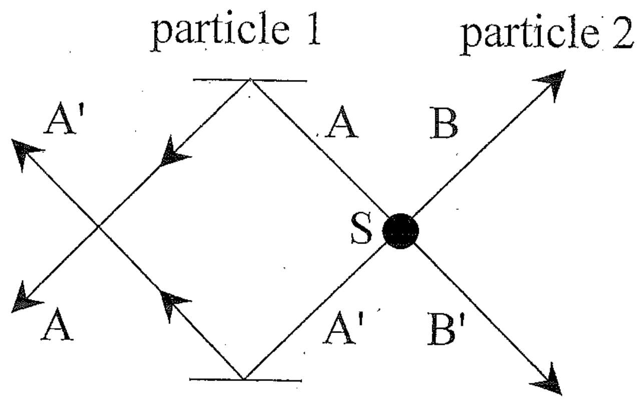
$$\dot{X}_1 = \frac{\partial}{\partial x_2} S(X_1, X_2, t) \ ,$$

which obviously depends on the position of the second particle. If we "act" on the second particle and thus move it, we instantaneously affect the behavior of the first particle.

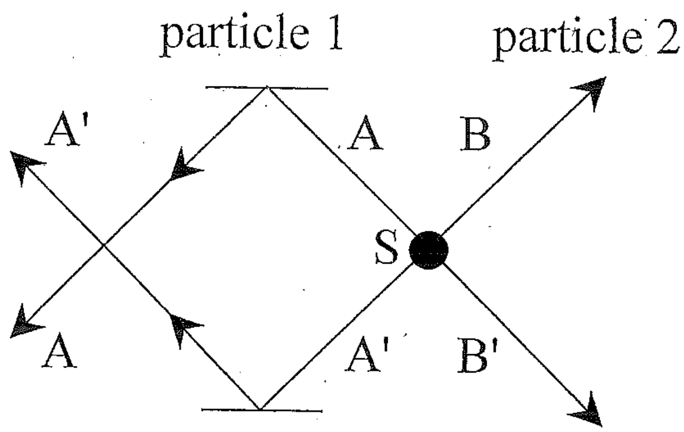


To understand this more intuitively, let us consider the following example, due to Dien Rice. The figure shows a two-particle quantum state, emitted from the source S, and which is a superposition of "particle 1 following path A and particle 2 following path B" and of "particle 1 following path A' and particle 2 following path B'":

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1A\rangle|2B\rangle - |1A'\rangle|2B'\rangle) \ .$$

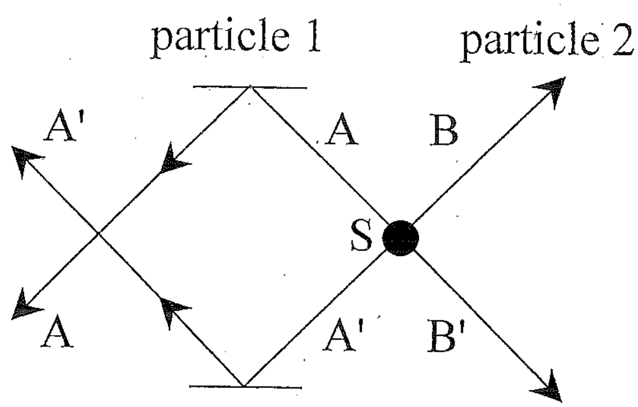


Of course, in ordinary quantum mechanics, the particles do not follow any path, so the paths here are just paths along which the wave function propagates.



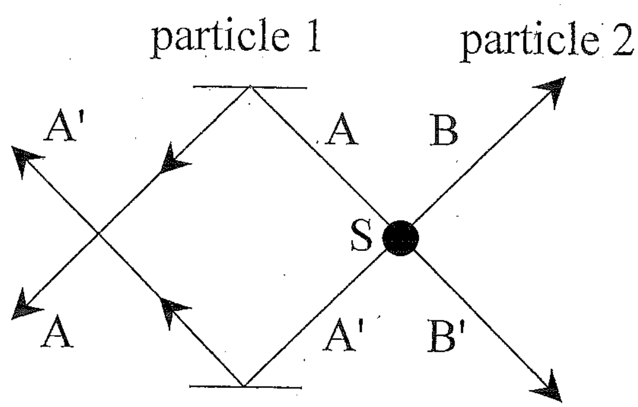
On the other hand, in the de Broglie–Bohm theory, the particles do follow paths, determined by the part of the wave function $|1A\rangle|2B\rangle$ or $|1A'\rangle|2B'\rangle$ whose support they happen to lie in.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1A\rangle|2B\rangle - |1A'\rangle|2B'\rangle) \ .$$



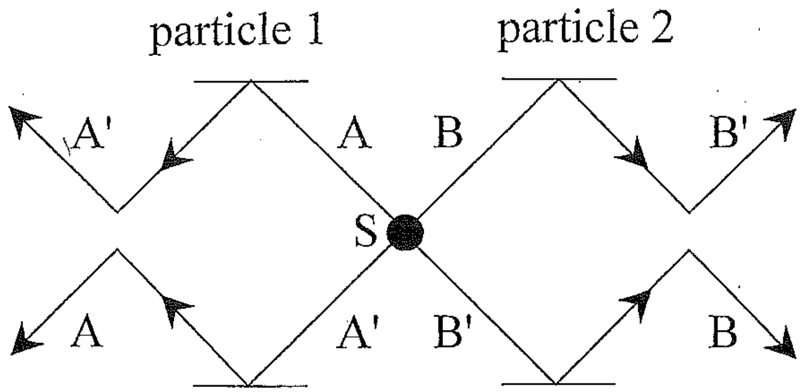
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1A\rangle|2B\rangle - |1A'\rangle|2B'\rangle) \ .$$

Here, there are mirrors along the possible paths followed by the wave function 1 that reflect it, so that the paths A and A' cross.



$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1A\rangle|2B\rangle - |1A'\rangle|2B'\rangle) \ .$$

Now, if we detect particle 2 along path B, this means that the support of the particles is in the part $|1A\rangle|2B\rangle$ of the wave function, and therefore that particle 1 will be found along path A, if detected (that is, at the bottom of the figure), and vice versa for paths B' and A'.



Let us now introduce more mirrors, this time along the possible paths followed by wave function 2.

To proceed with our argument, we need an important but elementary property of the de Broglie–Bohm evolution: the paths of the particles can never cross each other, at the same time, in the space $\mathbb{R}^4$ (each particle's motion takes place in a plane $\mathbb{R}^2$).

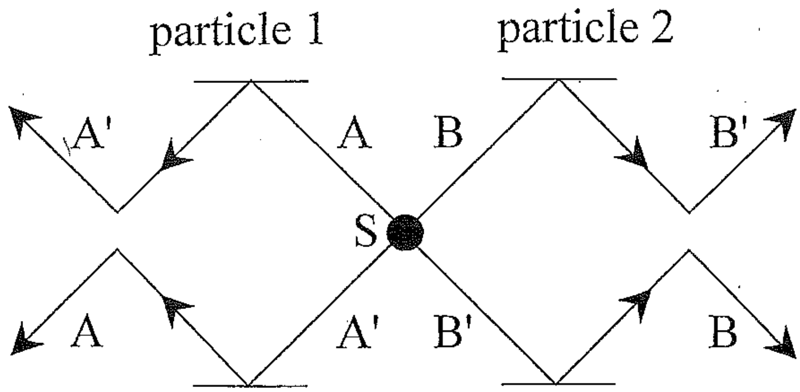
$$\dot{X}_1 = f(X_1, X_2, t),$$

$$\dot{X}_2 = g(X_1, X_2, t),$$

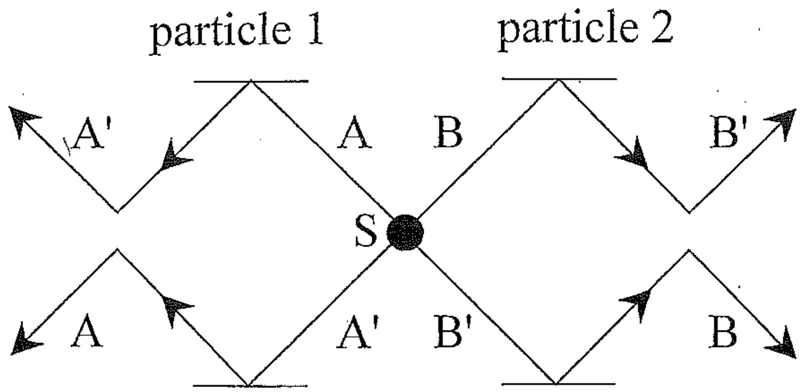
$X_i \in \mathbb{R}^2$.

No crossing of trajectories at the same time in $\mathbb{R}^4$!

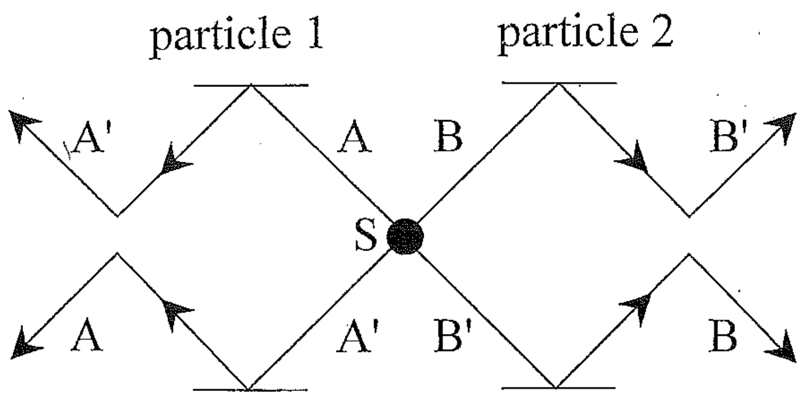
This is because the dynamics is a first order equation, and the velocities are uniquely determined by the positions, at any given time. Crossing of trajectories at the same time is therefore impossible, since it would mean that the same positions (at the point and at the time of crossing) could correspond to two different velocities (each pointing in the direction of one of the crossing paths).



But then, the only possibility is that the paths *followed by the particles* bounce off each other. What the arrows in the figure above now indicate are the possible particle paths. Note, however, that the paths A, A', B, B' of the *wave functions* cross each other, for A, A' and now symmetrically for B, B', contrary to the particle paths.

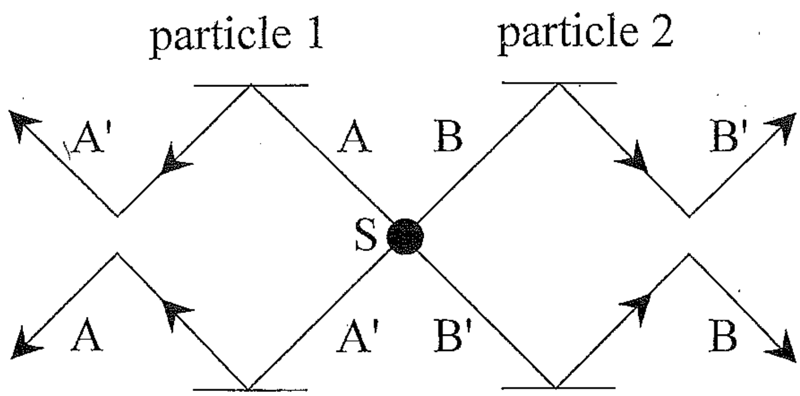


Thus, now, if particle 1 starts out in the support of the part A of the wave function, it will end up in the support of part A' and vice versa; and similarly for particle 2 and the support of parts B, B' of the wave function.

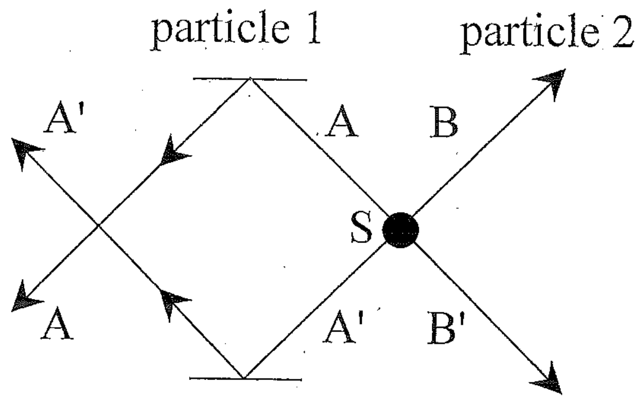


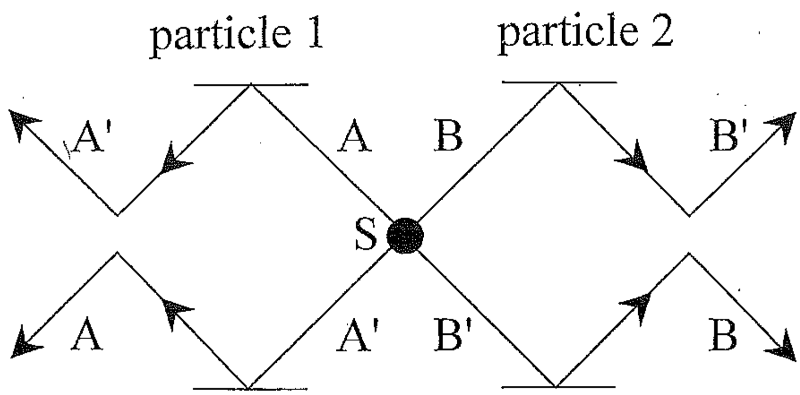
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1A\rangle|2B\rangle - |1A'\rangle|2B'\rangle) \ .$$

When the paths of the wave functions cross each other, the particles will go from being in the support of the part $|1A\rangle|2B\rangle$ of the quantum state to being in the support of the part $|1A'\rangle|2B'\rangle$.



The particles switch horses so to speak, i.e., they go from being guided by one of the wave functions to being guided by the other. So if the particle 2 is detected at the top of the figure, particle 1 will also be detected at the top of that figure (unlike what happened without the mirrors on the possible paths of particle 2).





This illustrates how non-locality works in the de Broglie–Bohm theory: by putting mirrors along the possible paths of particle 2, one instantaneously modifies the path followed by particle 1 since, without those mirrors, if particle 1 starts in the support of the part $|1A\rangle|2B\rangle$ of the quantum state, it will end at the bottom of the figure while if the mirrors are inserted along the possible paths of particle 2, it will end at the top.

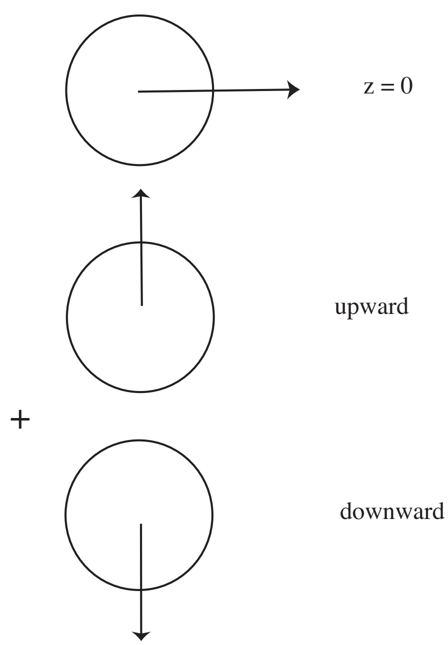
However, since this does not affect the statistics of what one detects, it cannot be used to send messages instantaneously: whatever is done to particle 2, one will detect particle 1 half of the time on each of the paths A or A'.

The fact that the de Broglie–Bohm theory is non-local is a quality rather than a defect, of course, since Bell showed that any correct theory must be non-local. Moreover, the non-locality is of the right type, i.e., just what is needed to reproduce Bell's results, but not more, where "more" might be a non-local theory allowing the propagation of messages.

WHAT ABOUT THE COLLAPSE OF THE QUANTUM STATE?

There is no REAL collapse, but an effective one. Remember what we said yesterday: if one starts with a state of the form:

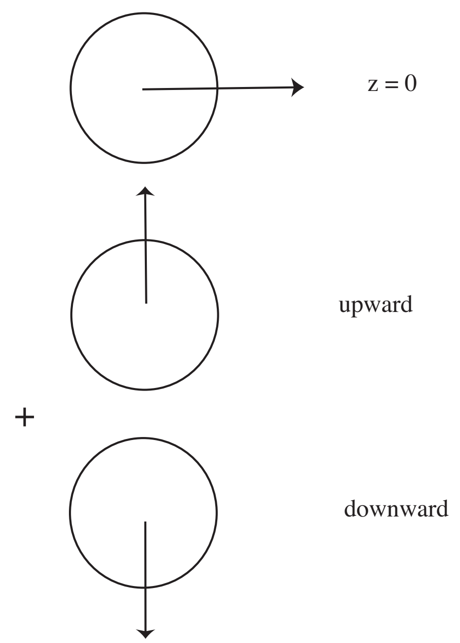
$\Psi_0 = \varphi_0(z)\left(c_1\begin{pmatrix}1\\0\end{pmatrix} + c_2\begin{pmatrix}0\\1\end{pmatrix}\right)$ where z is the coordinate of a macroscopic pointer with $\varphi_0(z)$ centered at $z = 0$.

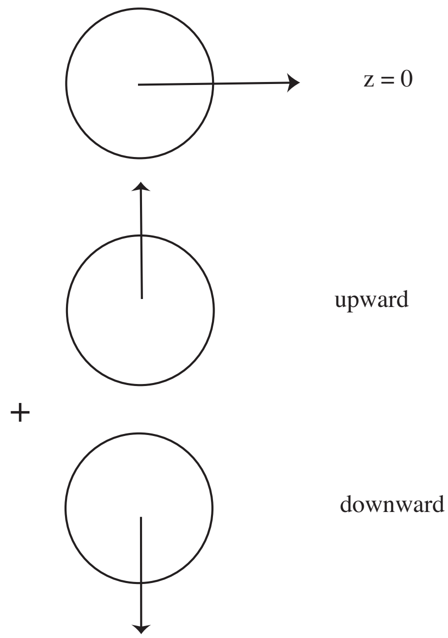


Since the evolution is *linear*, one obtains:

$$c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi_0(z-t) + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varphi_0(z+t)$$

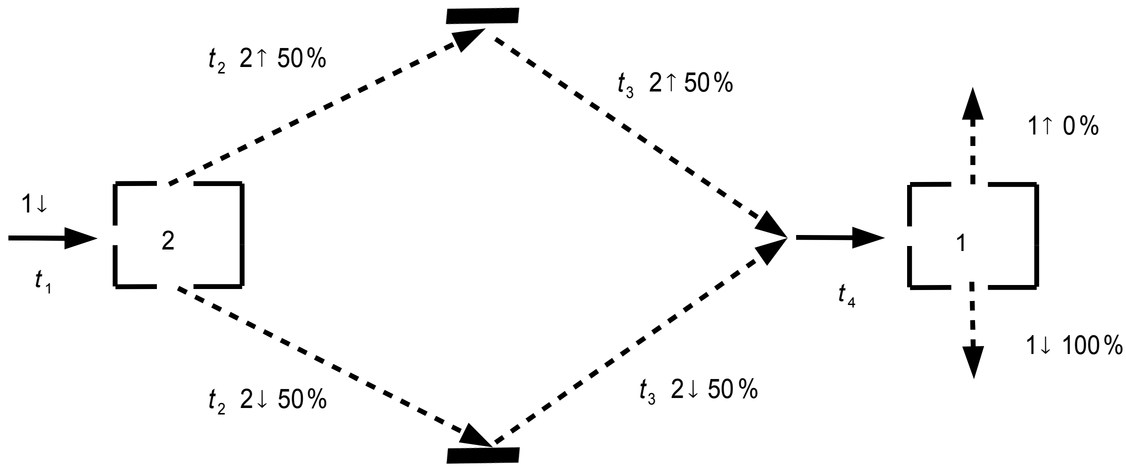
$\rightarrow \quad \varphi_0(z \pm t)$ centered at $z = \mp t$.

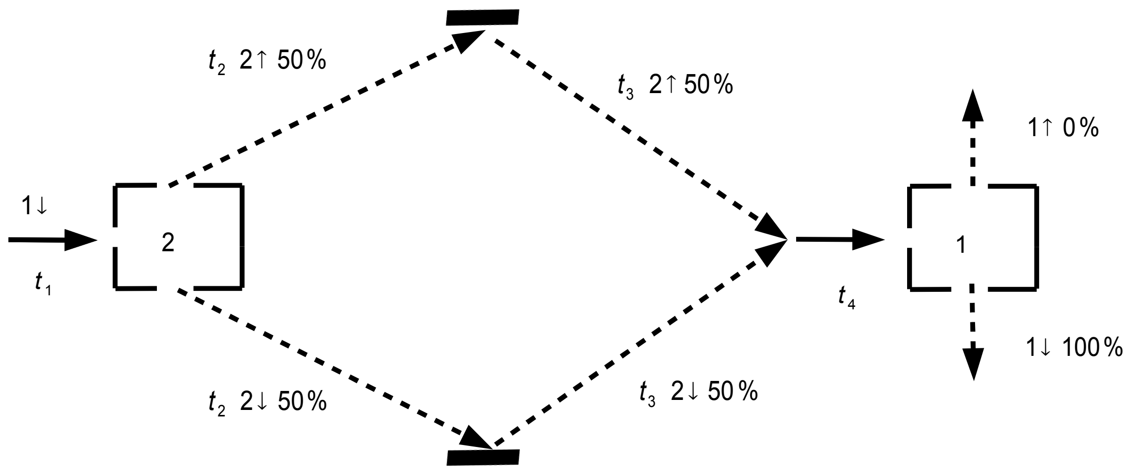




The quantum state of the pointer is in a superposition of two macroscopically distinct states, but the particles (the one being measured and those of the pointer) are in the support of ONE of those two states (which one depends on the initial conditions).

So, their future behavior will be guided by the quantum state in the support of which they are, and we can, in practice, "forget" about the other part, PROVIDED the two quantum states do NOT interfere in the future, as with the Mach-Zehnder interferometer:





BUT this sort of interference, in practice, impossible when one deals with many particles, such as all the particles in a pointer, or a cat, because one would have to make the two parts of the quantum state of *each* particle overlap again.

This impossibility is similar to the one of reversibility in classical statistical mechanics. So there is an "effective" or "in practice" reduction, but not an "in principle" one.

WHAT ABOUT RELATIVITY?

This is a real problem. BUT that is not a defect of the de Broglie-Bohm theory, but is due to non-locality.

We saw that, in the example of the mirrors, adding mirrors on one side affects instantaneously the situation on the other side (no matter how distant they are); But "instantaneous" is not a relativistic notion. So, it is not easy to make a relativistically invariant de Broglie-Bohm theory, in a natural way.

BUT THAT IS TRUE ALSO OF ORDINARY QUANTUM MECHANICS.

BUT ISN'T THERE A RELATIVISTIC QUANTUM MECHANICS, e.g. QUANTUM FIELD THEORY, THAT RECONCILES QUANTUM MECHANICS AND RELATIVITY?

Well, try to find somewhere a relativistic treatment of the reduction or collapse of the quantum state. You will not find one.

The reason is obvious; consider the state, used in the proof of Bell's inequality:

$$\frac{1}{\sqrt{2}}(|A\ 1\uparrow> |B\ 1\downarrow> -|A\ 1\downarrow> |B\ 1\uparrow>)$$

If one measures FIRST the spin in direction 1 at X, and one sees $\uparrow$, the state becomes $|A\ 1\uparrow> |B\ 1\downarrow>$. A later measurement of the spin at Y will yield $|\downarrow>$ with certainty.

$$\frac{1}{\sqrt{2}}(|A\ 1\uparrow> |B\ 1\downarrow> -|A\ 1\downarrow> |B\ 1\uparrow>)$$

BUT if one measures FIRST the spin in direction 1 at Y, and one sees $\uparrow$, the state becomes $|A\ 1\downarrow> |B\ 1\uparrow>$. A later measurement of the spin at X will yield $|\downarrow>$ with certainty.

So, it matters who does what first, i.e. who collapses the quantum state non-locally, at both places simultaneously. BUT “FIRST” is not a relativistic invariant notion. SO, there is no relativistic treatment of the “collapse”.

Of course, if the collapse was purely epistemic and not physical, then its non-local nature would not be a problem, but Bell has shown that this is impossible.

For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory

John Bell

What Is the Relationship Between the de Broglie–Bohm Theory and Ordinary Quantum Mechanics?

It is the same theory! Or, more precisely, the de Broglie–Bohm theory *is* a theory, while ordinary quantum mechanics is not. Indeed, quantum mechanics doesn't even pretend to be a theory, but rather claims to be an algorithm allowing us to compute "results of measurements".

Another way to say this is that ordinary quantum mechanics is the algorithm used to compute results of measurements that is *derived* from the de Broglie–Bohm theory.

One might also say that ordinary quantum mechanics is simply a truncated version of the de Broglie–Bohm theory: in ordinary quantum mechanics, one ignores the particle trajectories, but since the empirical predictions of the de Broglie–Bohm theory are statistical, and are the same as those of ordinary quantum mechanics, there are no practical consequences of that omission.

Thus, ordinary quantum mechanics is sufficient "for all practical purposes" (FAPP) to use Bell's expression.

But it is the de Broglie–Bohm theory that *explains* why ordinary quantum mechanics is sufficient FAPP, something that is true but mysterious without de Broglie–Bohm. In particular, both the collapse rule and the statistical nature of the predictions find a natural explanation within the de Broglie–Bohm theory, due firstly to the analysis of measurement as a purely physical mechanism, always governed by the Schrödinger equation, and secondly to the hypothesis of quantum equilibrium.

The problem created by ordinary quantum mechanics is that, since it is not a physical theory (i.e. a theory about what goes on in the world), while (sometimes) pretending to be one or while claiming that such a theory is impossible, or is not the goal of physics, it has led to the creation of entire libraries of confusions, bad philosophy and mysticism.

A BIT OF HISTORY

In his 1932 book *Mathematische Grundlagen der Quantenmechanik*, John von Neumann proved a (very weak) no hidden variables theorem and concluded from it that:

> It is therefore not, as is often assumed, a question of a reinterpretation of quantum mechanics — the present system of quantum mechanics would have to be objectively false, in order that another description of the elementary processes than the statistical one be possible.
>
> JOHN VON NEUMANN

And Max Born, in semi-popular lectures added:

> No concealed parameters can be introduced with the help of which the indeterministic description could be transformed into a deterministic one. Hence, if a future theory should be deterministic, it cannot be a modification of the present one but must be essentially different.
>
> M. BORN

Here is how BELL reacted:

> Having read this, I relegated the question to the back of my mind and got on with more practical things. But in 1952, I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one.

More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the ‘observer’, could be eliminated. Moreover, the essential idea was one that had been advanced already by de Broglie in 1927, in his ‘pilot wave’ picture.

But why then had Born not told me of this 'pilot wave'? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing 'impossibility' proofs, after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg, could produce no more devastating criticism of Bohm's version than to brand it as 'metaphysical' and 'ideological'? Why is the pilot wave picture ignored in text books?

Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?

J.S. BELL

CONCLUSION: MAXIMAL CONFUSION IN THE LITERATURE

— Einstein is supposed to have tried to show that quantum mechanics in incomplete and failed (Bohr replied successfully).

But EPR were posing a dilemma: either quantum mechanics is incomplete or the world is non-local. It is true that they rejected the second half, but neither Bohr nor his followers understood the problem.

— In 1927 de Broglie proposes a non-local theory that eliminates any reference to an "observer" . The theory is dismissed (on dubious grounds) and rediscovered in 1952 by Bohm.

— In between (in 1932), von Neumann claims to have shown that no hidden variables can be added to the quantum mechanical formalism; i.e. quantum mechanics is “complete”. People tend to believe his theorem, without looking at the assumptions. Moreover, a “hidden variable” theory had been proposed before by de Broglie.

— After 1952, Bell understand Bohm's theory but wonders whether one can "do better", i.e. reproduce the quantum mechanical predictions in a LOCAL theory without observers. He proves in 1964 that this is impossible, by showing that the "hidden variables" (the spin values) that EPR showed were necessary to save locality cannot exist.

— Bell's result is (as he himself emphasized) is an argument **in favor** of Bohm's theory, since it shows that one cannot avoid non-locality, which is moreover a rather natural feature in Bohm's theory.

— Bell's result is generally taken to mean that "hidden variable" theories cannot be compatible with the results of quantum mechanics, including Bohm's theory, which is compatible with those results, which is non-local (hence compatible with Bell's results) and explains why one cannot introduce those spin values that Bell shows are impossible.

Can anybody do WORSE?

MAYBE WE SHOULD TURN TO LITERATURE !

I know that most men, including those at ease with problems of the highest complexity, can seldom accept even the simplest and most obvious truth if it be such as would oblige them to admit the falsity of conclusions which they have delighted in explaining to colleagues, which they have proudly taught to others, and which they have woven, thread by thread, into the fabric of their lives.

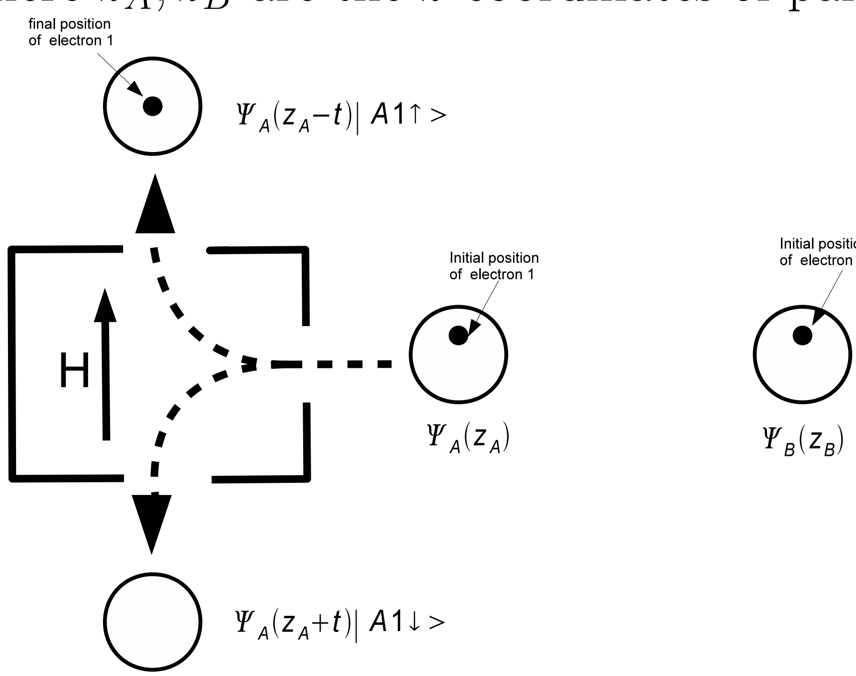
TOLSTOY

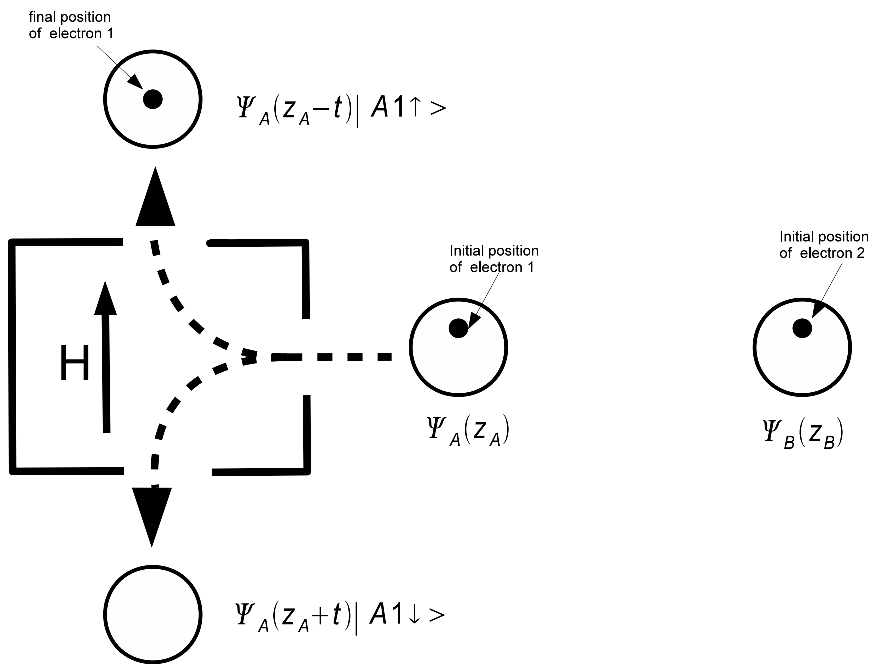
THE EPR–BELL EXPERIMENTS IN THE FRAMEWORK OF THE DE BROGLIE–BOHM THEORY

We start with a quantum state:

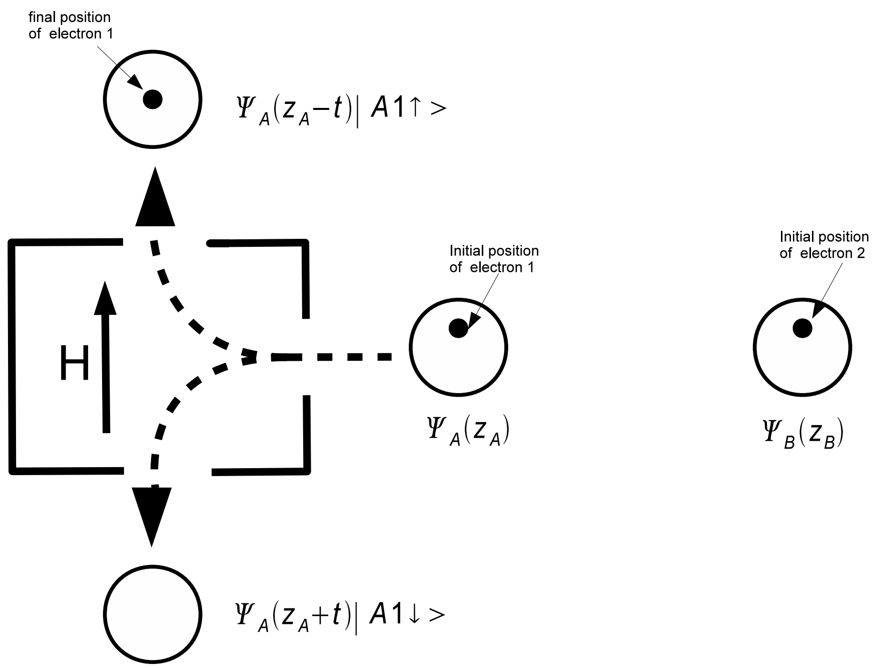
$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|A\,1\uparrow\rangle|B\,1\downarrow\rangle\Psi_A(z_A)\Psi_B(z_B)$$
$$-|A\,1\downarrow\rangle|B\,1\uparrow\rangle\Psi_A(z_A)\Psi_B(z_B)]\ ,$$

where z_A, z_B are the z coordinates of particles A and B.





If we measure first the spin of particle A in direction 1, we will get the up result, given the initial particle position, assuming that things are symmetric.



But this means that the quantum state becomes effectively

$$|A\ 1\uparrow\rangle|B\ 1\downarrow\rangle\Psi_A(z_A - t)\Psi_B(z_B),$$

since the particle A is in the support of that part of the state, and the other part

$$|A\ 1\downarrow\rangle|B\ 1\uparrow\rangle\Psi_A(z_A + t)\Psi_B(z_B)$$

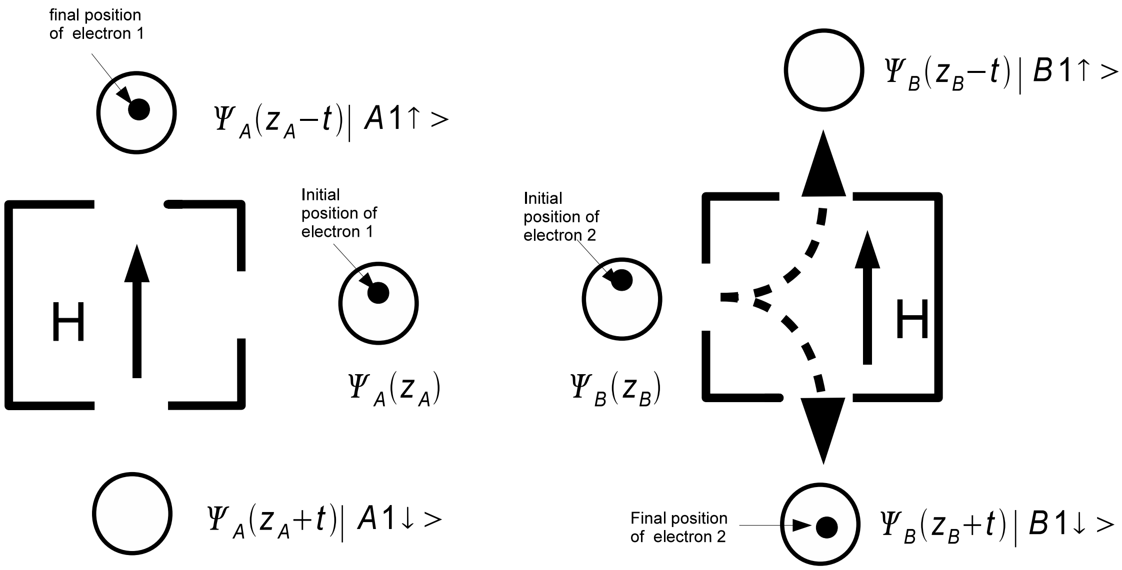
now has a support disjoint from

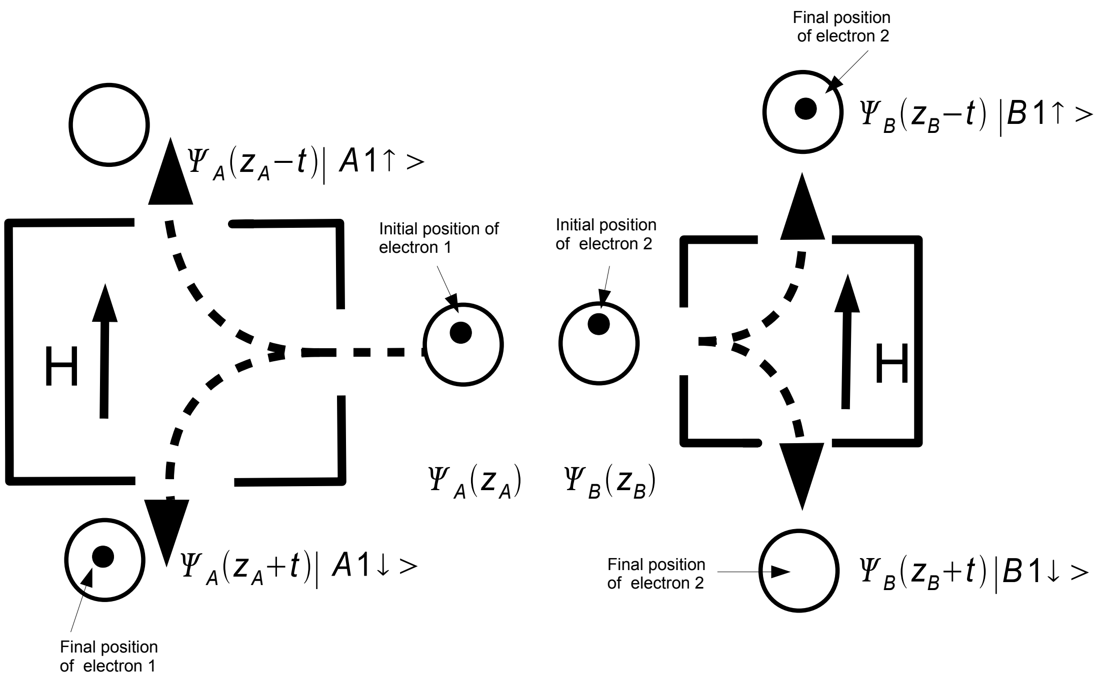
$$|A\ 1\uparrow\rangle|B\ 1\downarrow\rangle\Psi_A(z_A - t)\Psi_B(z_B).$$

This implies that particle B must also be guided only by that part of the wave function. Since the spin part of the quantum state is $|B\ 1\downarrow\rangle$, it will go down in all cases, irrespective of its initial position, if one measures its spin in direction 1, after the measurement at A.

The final quantum state for the two particles will be

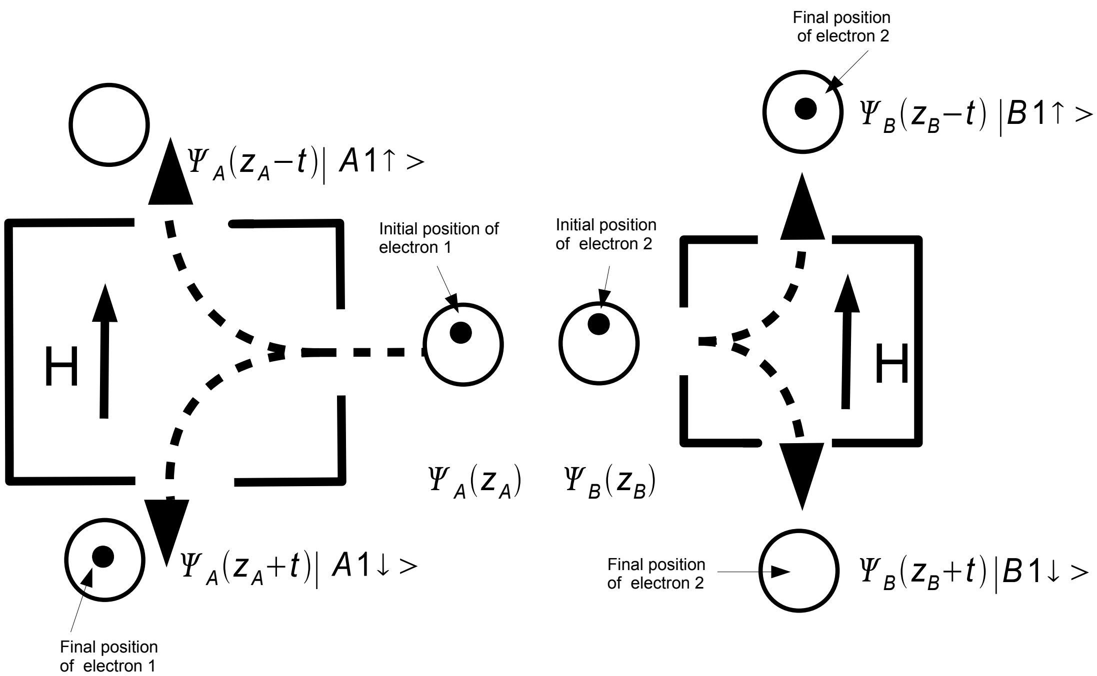
$$|A\ 1\uparrow\rangle|B\ 1\downarrow\rangle\Psi_A(z_A - t)\Psi_B(z_B + t).$$





Consider now what happens if one measures the spin of particle B first. Since the initial position is above the symmetry axis, particle B goes up and the quantum state of the two particles becomes

$$|A\ 1\downarrow\rangle|B\ 1\uparrow\rangle\Psi_A(z_A)\Psi_B(z_B - t).$$



But then, a later measurement of the spin of particle A in direction 1 yields the result down, again irrespective of the initial position of that particle, and the final quantum state for the two particles is

$$|A\ 1\downarrow\rangle|B\ 1\uparrow\rangle\Psi_A(z_A+t)\Psi_B(z_B-t).$$

All this illustrates how non-locality works in the de Broglie–Bohm theory. There is a genuine action at a distance here, since, if we measure the spin at A first, then with the initial particle positions above the nodal line, particle A will go up and particle B will go down, whereas if we measure the spin at B first, then particle B will go up, and particle A will go down.

However, in the de Broglie–Bohm theory, no message can be sent in the EPR–Bell type situations if we assume quantum equilibrium (just like with the mirrors), because then each side will see a “random” sequence of up and down results, whichever measurement one chooses to make, and there is no way to manipulate those measurements so as to transmit a message.