THE SECOND QUANTUM MYSTERY: NON-LOCALITTY

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WHAT WE SAW

$|\Psi|^2$ has a probabilistic interpretation

$|\Psi(x)|^2$ = density of probability of the particle "being observed" at x.

More generally, if

$$\Psi = \sum_i c_i \Psi_i,$$

where, for some self-adjoint operator A,

$$A\Psi_i = \lambda_i \Psi_i,$$

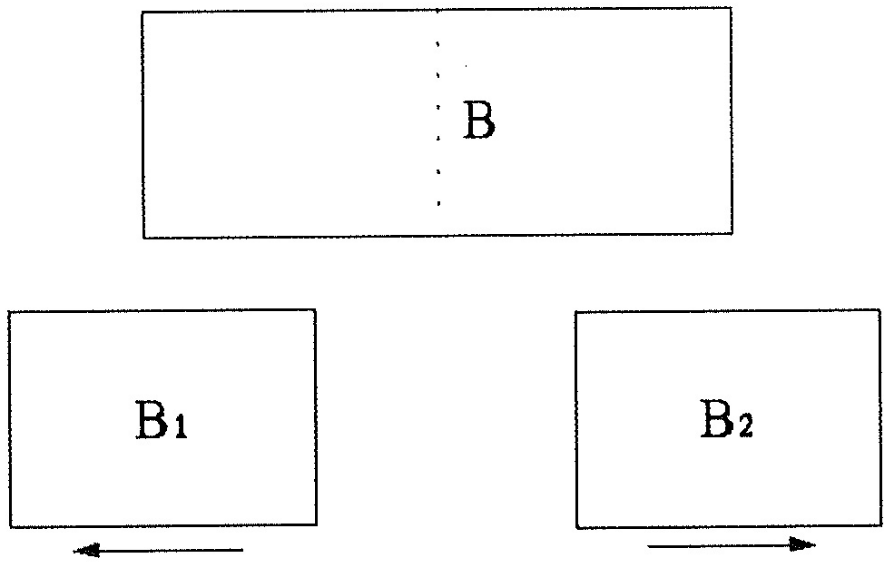
and the $\Psi_i' s$ form a basis of vectors, then the measurement of the "observable" associated with the operator A, will give the result λ_i with probability $|c_i|^2$.

But we KNOW, from the no hidden variable theorems, that one cannot consider measurements, in general, a revealing pre-existing properties of the system being "measured".

THIS HAS IMPLICATIONS CONCERNING THE POSSIBILITY OF ACTIONS AT A DISTANCE, OR NON-LOCALITY.

Let us consider a simple example:

Einstein's boxes



A single particle is in Box B.

The $|state>=|B>=\Psi(x)$, where $\Psi(x)$ is spread out through the box B.

One cuts the box in two half-boxes,

The state becomes

$\longrightarrow \frac{1}{\sqrt{2}}(|B_1>+|B_2>)$

where $|B_i>$ = particle "is" in box B_i, $i=1,2$.

The two half-boxes B_1 and B_2 are then separated and sent as far apart as one wants.

If one opens one of the boxes (say B_1) and that one does *not* find the particle, one *knows* that it is in B_2. Therefore, the state "collapses" instantaneously and in a non-local way, since the two boxes are as far part as one wants.

One opens box $B_1 \longrightarrow$ nothing

This is a "measurement", therefore state $\longrightarrow |B_2>$

(and, if one opens the box B_2, one will find the particle !).

DILEMMA:

Is the reduction or collapse of the

$|$ state $>$ a real (= physical) operation

or does it represent only our knowledge (= epistemic, as in classical probabilities) ?

<u>If</u> physical $\longrightarrow$ A non-local form of causality exists

<u>If</u> epistemic $\longrightarrow$ quantum mechanics "incomplete" : there exists other variables than the quantum state that describe the system.

These variables would tell in which half-box the particle **IS** *before one opens either of them.*

Of the two branches of the dilemma, incompleteness is by far the most reasonable one! That was actually Einstein's position, which is in general regarded as unreasonable.

However, it turns out that, putting aside for the moment the issue of completeness, one can *prove* non-locality.

What is non-locality ?

Non-local causality (causality NOT mere correlation)

Properties of non-local actions, if we assume that the reduction is physical:

1. Instantaneous: opening box B_1 instantaneously creates the particle in box B_2.

2. a. Extends arbitrarily far.

   b. The effect does not decrease with the distance.

3. Individuated (consider many pairs of half-boxes: opening one half-box affects the situation in the other half-box but nowhere else).

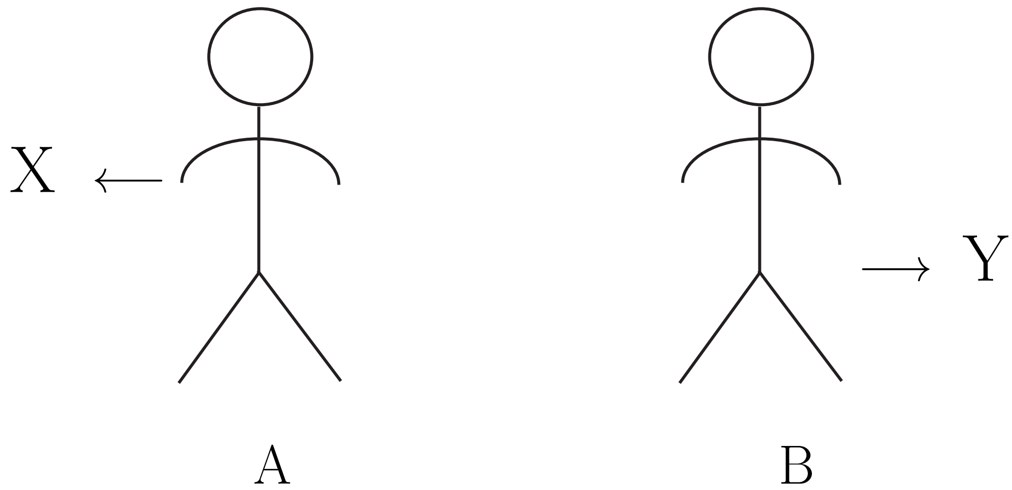
4. Cannot be used to transmit messages, if the results are "random".

Newton's gravity : 1, 2a and not 4

Post-Newtonian physics (e.g. field theories) : 2a but not 4

Is there a phenomenon with properties : 1-4 ?

HOW TO PROVE NON-LOCALITY ?



3 questions 1,2,3

2 answers yes/no

Questions and answers vary. But when the same question is asked at X and Y, one always gets the same answer.

DILEMMA : *EITHER* the answers are predetermined *OR* there exists a form of causality at a distance *after* one asks the questions.

TRY FOR YOURSELF TO FIND A THIRD POSSIBILITY.

This is the Einstein Podolsky and Rosen (EPR-1935) argument (in Bohm's formulation).

BUT

This assumption

(alone)

leads to a contradiction with observations made when the questions are different.

Bell (1964)

PROOF

3 Questions 1 2 3

2 Answers Yes/No

If the answers are given in advance, there exists $2^3 = 8$ possibilities :

1	2	3
Y	Y	Y
Y	Y	N
Y	N	Y
Y	N	N
N	Y	Y
N	Y	N
N	N	Y
N	N	N

In *each case* there are at least *two questions* with the same answer.

Therefore,

$$\begin{aligned}
&\quad \text{Frequency (answer to } 1 = \text{answer to } 2) \\
&+ \text{Frequency (answer to } 2 = \text{answer to } 3) \\
&+ \text{Frequency (answer to } 3 = \text{answer to } 1) \geq 1
\end{aligned}$$

BUT,

in some experiments,

$$\begin{aligned}
&\quad \text{Frequency (answer to } 1 = \text{answer to } 2) \\
&= \text{Frequency (answer to } 2 = \text{answer to } 3) \\
&= \text{Frequency (answer to } 3 = \text{answer to } 1) \\
&= \frac{1}{4}
\end{aligned}$$

$$\Rightarrow \frac{3}{4} \geq 1$$

FALSE !

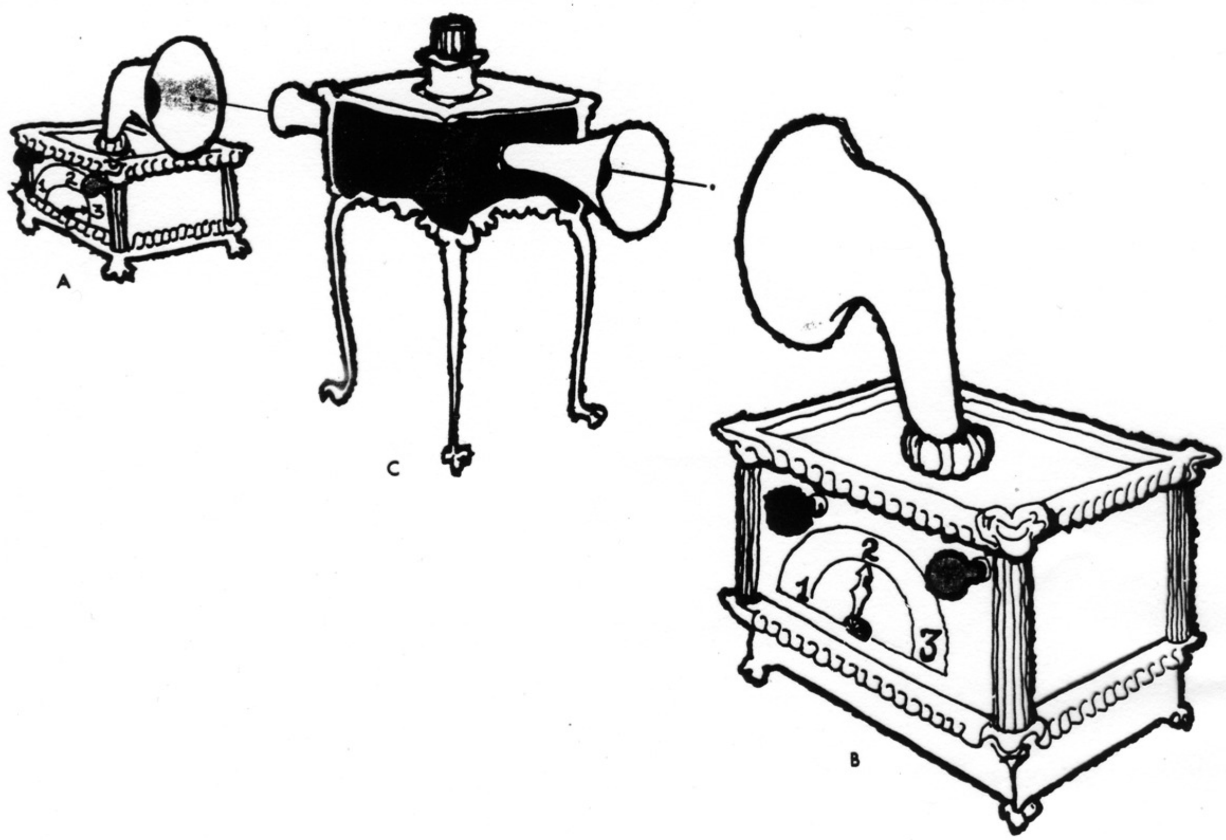
$\Rightarrow$ CONTRADICTION

That's all. That's the difficulty.

I've entertained myself always by squeezing the difficulty of quantum mechanics into a smaller and smaller place, so as to get more and more worried about this particular item. It seems to be almost ridiculous that you can squeeze it to a numerical question that one thing is bigger than another.

R. FEYNMAN, in "Simulating physics with computers" (1982)

EXPERIMENTS



1, 2, 3 =

QUESTIONS

(left/right)

Red

Green

≈ 2 ANSWERS

(Yes/No)

31RR	22GG	33GG
33GG	11RR	21GR
33RR	33GG	13GR
12GR	31GR	23GR
33GG	12GG	22RR
21GR	21GR	11RR
21RR	33GG	21GR
22RR	21RR	21RR
33GG	12GR	23GG
11GG	22RR	32GR
23RR	13RG	33RR
32GR	12RG	33GG
12GR	23GG	33GG
12RG	11GG	23GR
11GG	13RG	21GR
31RG	21RG	12RR
12RG	33RR	32GR
13GR	32GR	32GR
22GG	32GG	33GG
12RG	33GG	31RG
12GR	21RR	13RR
22GG	12RG	13RG
23GR	22GG	32RG
33RR	11GG	31GR
33GG	23GR	23RR
31RG	22RR	33RR
31RR	11GG	13GR
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22RR	21GG	12RG
12GG	23GR	23GR
33RR	22GG	32GR
11RR	22GG	21GG
23GG	31GG	21RG
23GG	13GR	13RG
33RR	21GR	13RG
23GR	33RR	13RG
21GG	23RR	13GR
13GR	22RR	23RG
33GG	12RR	22GG
11GG	23RG	11RR
12RR	23RG	31RG
12GG	32GR	23RR
31GG	31RG	23RG
32RG	22GG	11RR
21GR	11GG	32RG
22GG	11GG	32GR
22RR	21RG	13GG
13RR	11RR	23GR
21GG	12RG	32GR
23GR	23GR	22RR

35 out of ≈ 130

QUANTUM DESCRIPTION

(NOT needed, but most people insist)

AN IMPORTANT PRELIMINARY REMARK

Consider two charges, q_1 and q_2. Each of the charges produces an electric field $E_1(x)$ and $E_2(x)$, $x \in \mathbb{R}^3$.

The total electric field is $E(x) = E_1(x) + E_2(x)$. It is always defined on $\mathbb{R}^3$.

But, if we have two particles in quantum mechanics, the wave function is defined on $\mathbb{R}^6$:

$$\Psi = \Psi(x_1, x_2),$$

with $x_1, x_2 \in \mathbb{R}^3$. For N particles, Ψ is a function defined on $\mathbb{R}^{3N}$: *THE CONFIGURATION SPACE.*

That makes a lot of difference and is the source of non-locality.

QUANTUM DESCRIPTION

A and B are replaced by particles

X and Y are devices that "measure the spin" along some direction.

1, 2, 3 = 3 possible directions for that "measurement".

Yes/No = Up/Down.

$|$ state of the two particles $>$

$$= \frac{1}{\sqrt{2}}(|A\ 1\uparrow> |B\ 1\downarrow> - |A\ 1\downarrow> |B\ 1\uparrow>)$$
$$= \frac{1}{\sqrt{2}}(|A\ 2\uparrow> |B\ 2\downarrow> - |A\ 2\downarrow> |B\ 2\uparrow>)$$
$$= \frac{1}{\sqrt{2}}(|A\ 3\uparrow> |B\ 3\downarrow> - |A\ 3\downarrow> |B\ 3\uparrow>)$$

These three representations follow from rotation invariance (in the "spin space")-that is an elementary fact about quantum mechanics.

Let us consider one representation:

$| \text{ state of the two particles } >$

$= \frac{1}{\sqrt{2}}(|A\ 1 \uparrow> |B\ 1 \downarrow> - |A\ 1 \downarrow> |B\ 1 \uparrow>)$

If one measures the spin in direction 1 at X, and one sees $\uparrow$, the state becomes $|A\ 1 \uparrow> |B\ 1 \downarrow>$. A later measurement of the spin at Y will yield $| \downarrow>$ with certainty.

If one sees $\downarrow$, the state becomes $|A\ 1 \downarrow>\ \ |B\ 1 \uparrow>$ and a later measurement of the spin at Y will yield $| \uparrow>$ with certainty.

Similar result if one measures the spin in direction 2 or 3 at X.

But then the state changes *non-locally* at Y.

Same dilemma as for Einstein's boxes :

reduction of the $\mid$ state $>$ = physical or epistemic ?

<u>If</u> physical $\longrightarrow$ non-locality

<u>If</u> epistemic $\longrightarrow$ "answers" are given in advance, i.e. the particle B *is* $1 \uparrow$ or $1 \downarrow$, $2 \uparrow$ or $2 \downarrow$, $3 \uparrow$ or $3 \downarrow$, *before* any measurement at X. These answers would be "hidden variables".

BUT (Bell 1964) this leads to a contradiction with observations made when the directions in which the spin is "measured" are *different* at X and Y (the 1/4 is the result of standard quantum mechanical computations).

NO ASSUMPTION OF DETERMINISM, "REALISM" OR HIDDEN VARIABLES.

This is sometimes called a no hidden variable result, because it shows that one cannot introduce those pre-existing answers (the spin values) that would "save " locality. But the significance of the result is that, combined with the EPR argument, it refutes locality, not merely that it rejects (certain) "hidden variables".

To summarize: the perfect correlations (here, we have perfect anti-correlations, but that is a matter of conventions for YES/NO) are not merely correlations, but the result of a subtle form of non-locality. In other words, the reduction of the quantum state, which is non-local, is not merely epistemic, but related to something physical.

Where does the $1/4$ come from?

Let us compute $\mathbf{E_{a,b}} \equiv < \Psi|\sigma^A_{\mathbf{a}} \otimes \sigma^B_{\mathbf{b}}|\Psi >$, where $\mathbf{a}$, $\mathbf{b}$ are unit vectors in directions (1, 2 ou 3) along which one "measures" the spin at A or B.

This quantity is bilinear in $\mathbf{a}$, $\mathbf{b}$ and rotation invariant, thus of the form $\lambda\mathbf{a} \cdot \mathbf{b}$, for a certain $\lambda \in \mathbf{R}$.

Taking $\mathbf{a} = \mathbf{b}$, and because of perfect anti-correlations, one finds $\lambda = -1$, et thus $\mathbf{E_{a,b}} = -\cos\theta$, where θ is the angle between the directions $\mathbf{a}$ and $\mathbf{b}$. Since $v_A(\mathbf{a}), v_B(\mathbf{b}) = \pm 1$, $P(v_A(\mathbf{a}) = -v_B(\mathbf{b})) = \frac{1-\mathbf{E_{a,b}}}{2} = \frac{1+\cos\theta}{2}$.

One then chooses the directions:

$1 \Rightarrow 0$ degree,

$2 \Rightarrow 120$ degrees,

$3 \Rightarrow 240$ degrees.

(in each case $\cos\theta = \cos 120 = -1/2$ and $\frac{1+\cos\theta}{2} = 1/4$).

Finally, one changes conventions at A and B: $v_A(\mathbf{a}) = +1$ equals "yes", $v_A(\mathbf{a}) = -1$ equals "no", but $v_B(\mathbf{b}) = +1$ equals "no" $v_A(\mathbf{b}) = -1$ equals "yes".

IS THERE ANY WAY TO AVOID THE CONCLUSION ABOUT NON-LOCALITY?

> You might shrug your shoulders and say 'coincidences happen all the time', or 'that's life'. Such an attitude is indeed sometimes advocated by otherwise serious people in the context of quantum philosophy. But outside that peculiar context, such an attitude would be dismissed as unscientific. The scientific attitude is that correlations cry out for explanation.
>
> John Bell

A variant of the “shrugging one’s shoulders” argument, is to invoke a sort of “conspiracy”: for example, that each person has an answer to only one question but that, each time, and no matter how many times the experiment is repeated, that happens to be the question that is being asked to him or her. If we make that assumption, then our theorem cannot be derived (for the proof of the theorem to work, we need to assume pre-existing answers for three questions).

Another suggestion sometimes made is that the ordinary rules of probability do not apply in the EPR–Bell situation. But, since the reasoning here relies only on frequencies of results of experiments, and since the latter obviously do satisfy the ordinary rules of probability, this attempt to “save locality”, by trying to deny the implications of the EPR–Bell argument, does not work.

One cannot use this to send messages

If one could, then relativity implies that one could send messages into one's own past.

— Each side sees a perfectly random sequence of YES/NO

— BUT if each person tells the other which "measurements" have been made (1, 2 or 3), then, they both know which result has been obtained on the other side when the same measurement is made on both sides.

$\Rightarrow$ Then, they both share a common sequence of YES/NO, which is form of "information". Since that information cannot possibly come from the source (Bell), some sort of non-local transmission of information has taken place.

This is the basis of quantum information theory $\rightarrow$ may lead to a better understanding of non-locality.

BELL WAS QUITE EXPLICIT ABOUT WHAT THIS MEANS

Let me summarize once again the logic that leads to the impasse. The EPRB correlations are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel. If we do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting. But this has implications for non-parallel settings which conflict with those of quantum mechanics. So we cannot dismiss intervention on one side as a *causal* influence on the other.

J. BELL

BUT BELL WAS WIDELY MISUNDERSTOOD

Bell was also conscious of the misunderstandings of his results : "It is important to note that to the limited degree to which determinism plays a role in the EPR argument, it is not assumed but inferred. What is held sacred is the principle of "local causality" - or "no action at a distance" ... It is remarkably difficult to get this point across, that determinism is not a presupposition of the analysis." And he added, unfortunately only in a footnote: "My own first paper on this subject (*Physics* **1**, 195 (1965)) starts with a summary of the EPR argument *from locality to* deterministic hidden variables. But the commentators have almost universally reported that it begins with deterministic hidden variables."

One example of such a commentator is Murray Gell-Mann:

> Some theoretical work of John Bell revealed that the EPRB experimental setup could be used to distinguish quantum mechanics from hypothetical hidden variable theories... After the publication of Bell's work, various teams of experimental physicists carried out the EPRB experiment. The result was eagerly awaited, although virtually all physicists were betting on the correctness of quantum mechanics, which was, in fact, vindicated by the outcome.
>
> M. GELL-MANN

The proof he [von Neumann] published... though it was made much more convincing later on by Kochen and Specker, still uses assumptions which, in my opinion, can quite reasonably be questioned... In my opinion, the most convincing argument against the theory of hidden variables was presented by J.S. Bell.

E. WIGNER

EINSTEIN WAS ALSO MISUNDERSTOOD

An essential aspect of this arrangement of things [physical objects] in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects "are situated in different parts of space". The following idea characterizes the relative independence of objects far apart in space (A and B) : external influence on A has no direct influence on B.

A. EINSTEIN

Here is how Born "understood" Einstein:

The root of the difference between Einstein and me was the axiom that events which happens in different places A and B are independent of one another, in the sense that an observation on the states of affairs at B cannot teach us anything about the state of affairs at A.

M. BORN

Bell comments this passage as follows:

"Misunderstanding could hardly be more complete. Einstein had no difficulty accepting that affairs in different places could be correlated. What he could not accept was that an intervention at one place could influence, immediately, affairs at the other."

Physicist David Mermin has an amusing summary of the situation:

Contemporary physicists come in two varieties. Type 1 physicists are bothered by EPR and Bell's theorem. Type 2 (the majority) are not, but one has to distinguish two subvarieties. Type 2a physicists explain why they are not bothered. Their explanations tend either to miss the point entirely (like Born's to Einstein) or to contain physical assertions that can be shown to be false. Type 2b are not bothered and refuse to explain why. Their position is unassailable. (There is a variant of type 2b who say that Bohr straightened out the whole business, but refuse to explain how.)

D. MERMIN

Yet, the same David Mermin also wrote:

“Bell’s theorem establishes that the value assigned to an observable must depend on the complete experimental arrangement under which it is measured, even when two arrangements differ only far from the region in which the value is ascertained – a fact that Bohm theory exemplifies, and that is now understood to be an unavoidable feature of any hidden-variables theory.

To those for whom nonlocality is anathema, Bell’s Theorem finally spells the death of the hidden-variables program.”

WHAT TO SAY?