KTH/CSC

Aalto University
School of Science
and Technology

A global view of quantum computation with noisy components

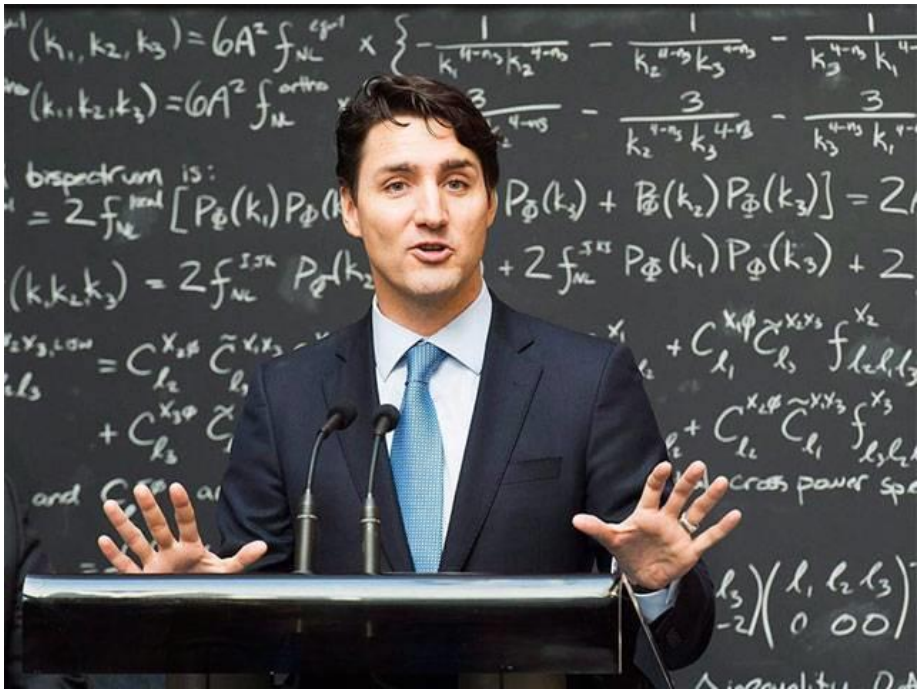
Erik Aurell

based on arXiv:1606.09407 (2016)

KTH/CSC

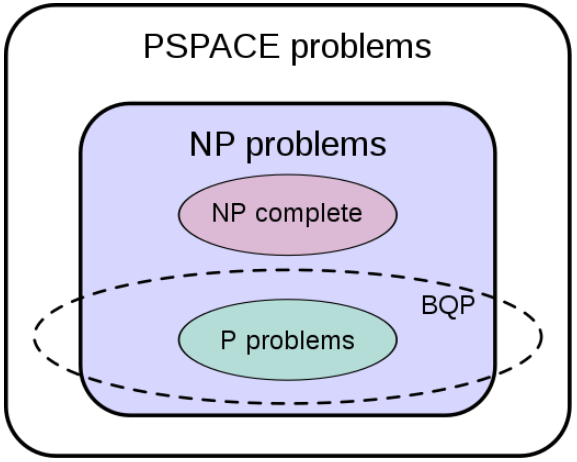
Aalto University
School of Science
and Technology

What is quantum computing?



Even (some) politicians know it is the use of superpositions to perform calculations that would be difficult to perform with a classical computer.

Some well-known results such as Shor's algorithm to factorize integers into primes. Not believed to solve all NP-hard problems.



Conjecture *(wikipedia)*

My original motivation

KTH/CSC

Aalto University
School of Science
and Technology

Gil Kalai on quantum computing

<kaski Petteri <petteri.kaski@aalto.fi>
må 2016-04-25 10:27
Inbox

Till: Alava Mikko <mikko.alava@aalto.fi>; Erik Aurell;

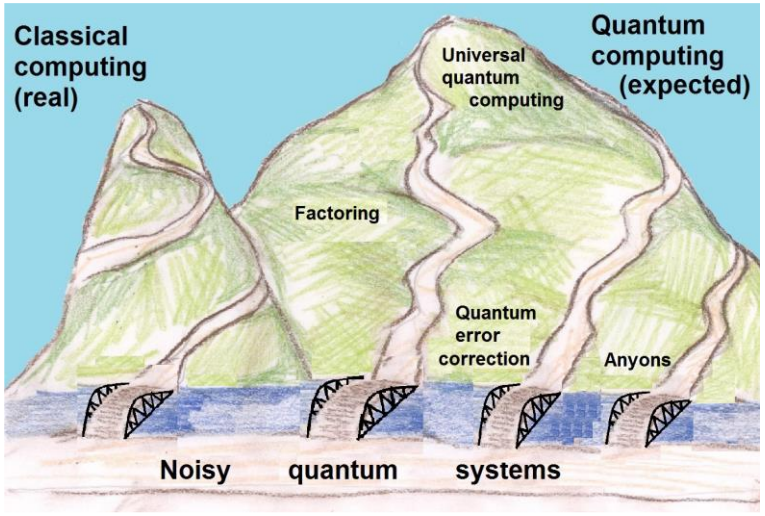
Du svarade den 2016-05-22 20:38.

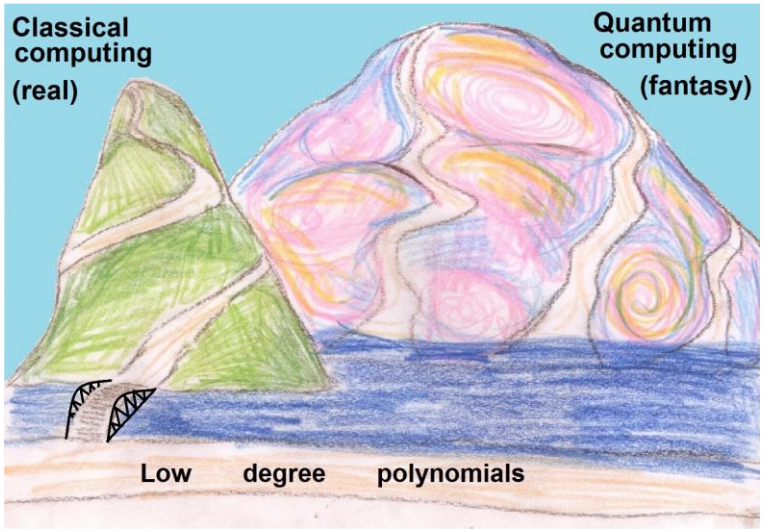
1 bilaga

(A shortened version appears in the May 2016 issue of Notices of the AMS.)

http://www.ams.org/publications/journals/notices/201605/rnoti-p508.pdf

P.





G. Kalai, *Notices of the AMS* **63**, 508-516 (2016)

Outline

Background material

- Notation and the theoretical baseline
- The toric code
- Correction of errors and error models

What's new

- Feynman-Vernon theory and path integral for spins
- Estimating the errors of computation using the toric code when the physical spins are linearly coupled to a bath of harmonic oscillators

KTH/CSC

Aalto University
School of Science
and Technology

Qubit notation: states and operations

A qubit is a 2-state system $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ $|\alpha|^2 + |\beta|^2 = 1$

A physical state is a ray $\alpha \leftrightarrow e^{i\varphi}\alpha \rightarrow 2$ real parameters

A pure state of N qubits is a ray in Hilbert space of dimension $\mathcal{N}=2^N$ $|\psi\rangle = \sum_i \alpha_i |i\rangle$ $|i\rangle = |i_1, i_2, \ldots, i_N\rangle$ $\mathcal{N}$-2 real parameters

A mixed state of N qubits is a density matrix, a Hermitian positive operator of unit trace ρ $\mathcal{N}^2$ -1 real parameters

A unitary evolution of N qubits U $\mathcal{N}^2$ -1 real parameters

$\psi \rightarrow U\psi \qquad \rho \rightarrow U\rho U^{-1}$

A general evolution of N qubits Φ $\mathcal{N}^4$ - $\mathcal{N}^2$ real parameters

KTH/CSC

Recalling Kraus' representation

Aalto University
School of Science
and Technology

A quantum operation is linear, and can hence be represented by a matrix. The "dynamical matrix" is linear transform of that matrix.

Sudarshan, Mathews & Rau, *Phys. Rev.* **121**: 920 (1961)
Choi, *Linear Alg. Appl.* **10**: 285 (1975)

$$\rho'_{\mu\nu} = L_{\mu\nu,mn}\rho_{mn} \qquad D_{\mu m,\nu n} = L_{\mu\nu,mn} \qquad \left\langle X \left| \rho'_{\mu\nu} \right| X \right\rangle \geq 0 \Rightarrow \left\langle x \otimes y \left| D \right| x \otimes y \right\rangle \geq 0$$

$$\rho'_{\mu\nu} = \left(\rho'\right)^{+}_{\mu\nu} \Rightarrow D_{\mu m,\nu n} = \left(D_{\nu n,\mu m}\right)^{*} \qquad \mathrm{Tr}\left[\rho'\right] = 1 \Rightarrow \sum_{\mu} D_{\mu m,\mu n} = \mathbf{1}_{mn}$$

D is positive, Hermitian of dimension $\mathcal{N}^2$, and obeys $\mathcal{N}^2$ constraints.

$$D_{\mu m,\nu n} = \sum_{i=1}^{\mathcal{N}^2} d_i \chi^{(i)}_{\mu m} \overline{\chi}^{(i)}_{\nu n} \qquad \rho'_{\mu\nu} = D_{\mu m,\nu n}\rho_{mn} = \sum_{i=1}^{\mathcal{N}^2} d_i \chi^{(i)}_{\mu m} \rho_{mn} \left(\chi^{(i)}\right)^{+}_{n\nu}$$

Coefficients d_i are non-negative, matrices $\chi^{(i)}$ are not unitary.

The theoretical baseline

KTH/CSC

Aalto University
School of Science
and Technology

pure initial state on N qubits	$\lvert i\rangle = \lvert i_1, i_2, \ldots, i_N\rangle \quad \rho_i = \lvert i\rangle\langle i\rvert$
unitary quantum operation	$\rho_f^{(0)} = \Phi_0[\rho_i] \quad \Phi_0[\rho] = U\rho U^{-1}$
measurement outcome	$P_{if}^{(0)} = \langle f \lvert \rho_f^{(0)} \rvert f\rangle$ where $\lvert f\rangle = \lvert f_1, f_2, \ldots, f_N\rangle$
non-unitary quantum operation	$\rho_f = \Phi[\rho_i] \quad P_{if} = \langle f \lvert \rho_f \rvert f\rangle$
total variational distance	$\mathrm{TVD} = \sum_f \lvert P_{if} - P_{if}^{(0)}\rvert$

If $\Phi = \Phi_1\Phi_2\cdots\Phi_L$ and $\Phi^{(0)} = \Phi_1^{(0)}\Phi_2^{(0)}\cdots\Phi_L^{(0)}$ and $\lvert\Phi_k - \Phi_k^{(0)}\rvert \le \varepsilon\ \forall k$,

and if each pair $(\Phi_k^{(0)}, \Phi_k)$ is a unitary and a general quantum gate, and

if clever choices are made for the norms $\lvert\cdot\rvert$ on states and operations, then

$$\mathrm{TVD} \le L \cdot \varepsilon$$

Aharonov, Kitaev & Nisan, STOC '98

KTH/CSC

Aalto University
School of Science
and Technology

Something more than AKN98 is needed

A quantum computer where the errors increase with the size of the computer is not computationally more powerful than a classical device. To do better however means to maintain coherence of a large quantum system for long times.

> "..quantum coherence is exceedingly sensitive to the unavoidable coupling with the environment…
>
> Manipulating such a quantum monster would be a feat almost as difficult as keeping Schrödinger's famous cat in a superposition of its dead and alive states"

S. Haroche, J-M. Raimond « Quantum computing: dream or nightmare?"
Physics Today August 1996, pp 51-52

Quantum error correction

Ingenious schemes for getting around the decoherence problem have recently been put forward. They rely on a variety of "watchdog" strategies […]

Haroche & Raimond (1996) *op cit*

QEC relies on special code spaces C. A *recovery operator* $\mathcal{R}\rho = \sum R_a \rho R_a^\dagger$ corrects a quantum operation $\Phi\rho = \sum A_i \rho A_i^\dagger$ with complete *fidelity* if $\forall x \in C \; \langle x|\mathcal{R}\Phi(|x\rangle\langle x|)|x\rangle = 1$. This leads to

$$P_C R_a A_i P_C = \lambda_{ai} P_C; \quad \text{where } \lambda_{ai} \text{ are constants, and to}$$

$$\forall x, y \in C : \langle x | A_i^+ A_j | y \rangle = \langle x | A_i^+ \sum_a R_a^+ R_a A_j | y \rangle = \sum_a \overline{\lambda}_{ia} \lambda_{ja} \langle x | y \rangle$$

Theorems 3.1 & 3.2 Knill, Laflamme, *Phys. Rev. A* **55**:900 (1997)

In general, the Knill-Laflamme conditions are not very constructive as to the choice of C, relative to a given Φ, and quite restrictive.

KTH/CSC

More error correction

Aalto University
School of Science
and Technology

The general representation of quantum evolution

$$\Phi\rho = \sum_{i=1}^{\mathcal{N}^2} A_i \rho A_i^+; \qquad \sum_{i=1}^{\mathcal{N}^2} A_i^+ A_i = \mathbf{1}$$

Sudarshan et al, *Phys. Rev.* **121**: 920 (1961)
Choi, *Linear Alg. Appl.* **10**: 285 (1975)

The general condition for successful error correction (roughly) means that the operators $\{A_i\}$ must behave nicely on a code space C

Knill, Laflamme, *Phys. Rev. A* **55**: 900 (1997)

To construct general error recovery operators is a complex problem already for general 2-qubit maps and general rank-2 maps, see

Majgier, Maassen & Życzkowski *Quantum Inf Process* **9**:343 (2010)
Lipka-Bartosik & Życzkowski *Quantum Inf Process* **16**:9 (2017)

Nevertheless, we'll go to far more complicated systems…

Toric code / surface code

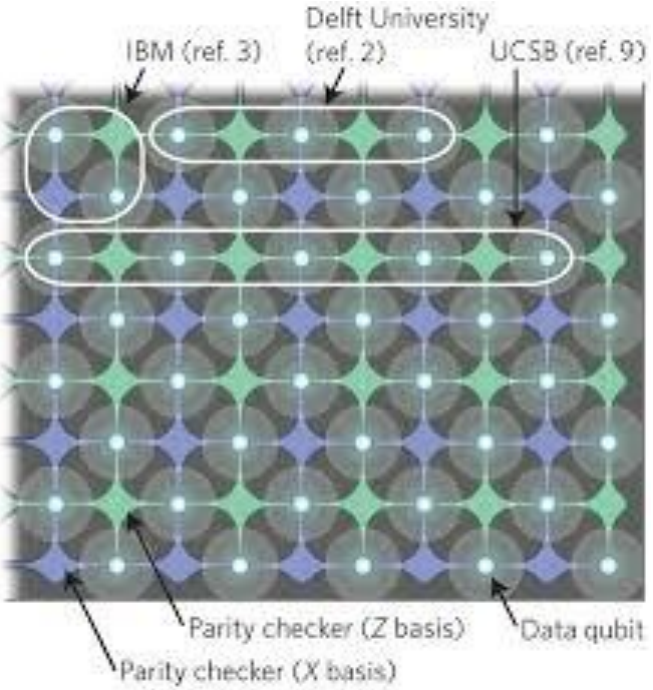
KTH/CSC

Aalto University
School of Science
and Technology

Surface codes work with physical spins on a 2D lattice. Many combinations of *z*- and *x*-components of four spins are measured continuously. The remaining degrees of freedom are extended states. For codes on a torus these are two *logical qubits*.

A. Kitaev, *Russian Math. Surv.* **52**, 1191-1249 (1997)
A. Fowler et al, *Phys. Rev. A* **86**, 032324 (2012)

S. Benjamin & J. Kelly, *Nature Materials* (2015)



A quantum computation interacting with a bath as in the spin-boson model could then be described by a Hamiltonian….

logical qubits

physical spins

$$H = \sum_{\text{gates } g} c_g \hat{\sigma}_{i_{g,1}}^{\alpha_{g,1}} \cdots \hat{\sigma}_{i_{g,k_g}}^{\alpha_{g,k_g}} + \sum_{\text{bath } b} \left[\sum_{\text{spins } i} C_{bi} x_b \hat{S}_i^z \right] + H_B$$

…where also the following operators, *stabilizers*, are measured continuously:

$$A_s = \Pi_{j \in \text{star}(s)} \hat{S}_j^x \quad B_p = \Pi_{j \in \partial p} \hat{S}_j^z$$

KTH/CSC

Aalto University
School of Science
and Technology

Error correction and accuracy threshold

Assume the state is initially pure, in an eigenspace of the stabilizers.

In the DKLP error model different spins are supposed to behave as if operators $\hat{\sigma}_x$ and $\hat{\sigma}_z$ were applied randomly, each with probability p. The density matrix of one spin then would then change as

$$\rho' = (1-p)^2 \rho + p(1-p)\hat{\sigma}_x \rho \hat{\sigma}_x + p(1-p)\hat{\sigma}_z \rho \hat{\sigma}_z + p^2 \hat{\sigma}_y \rho \hat{\sigma}_y$$

Denis, Kitaev, Landahl & Preskill *J. Math. Phys.* **43**: 4452-4505 (2002)
Terhal, *Rev. Mod. Phys.* **87**: 307 (2015)

Measuring the stabilizers gives indications of which error could have happened. Recovery is possible (under many assumptions) if $p < p_c$ where $p_c \approx 0.1094...$

The above has given rise to a large literature...

General-errors objection

The DKLP error model is a special case of the *random external field model*

$$\Phi^{REF}\rho = \sum_i p_i V_i \rho V_i^+; \quad V_i \text{ unitary and } \sum_i p_i = 1 \qquad \left(\hat{\sigma}_x = \tfrac{1}{i} e^{i\frac{\pi}{2}\hat{\sigma}_x}, \text{etc}\right)$$

The error mechanism modelled by DKLP is hence that of a classical uncertainty, as to which operator was applied (or not applied) on the system.

The dimensionality of random external field models is $\mathcal{N}^2$-1. That is only an $\mathcal{N}^{-2}$ fraction of all quantum operations. For 1-qubit ($\mathcal{N}$=2) the REF class is unitarily equivalent to *unital maps*, and the same as *Pauli channels*

$$\Phi^{Unital}\mathbf{1} = \mathbf{1} \qquad \Phi^{Pauli}\rho = p_0\rho + p_1\hat{\sigma}_x\rho\hat{\sigma}_x + p_2\hat{\sigma}_y\rho\hat{\sigma}_y + p_3\hat{\sigma}_z\rho\hat{\sigma}_z$$

The issue with general errors is not only if they can be detected by the stabilizers (perhaps they can). The more important issue is if (and how) they can also be corrected for: for sure they cannot by applying an external correcting field.

Markov objection

Markovian development of the system (quantum semigroup) takes place if either (A) bath temperature is high enough, or (B) coupling between system and bath is weak, and one rescales the time.

(A) is a problem if one needs a steady stream of pure state (cold) ancillae. (B) is a problem if there is also a time scale of the system dynamics.

Alicki, Lidar & Zanardi, *Phys. Rev. A* **73**, 052311 (2006)

On the related question if stabilizer codes can function as a quantum memory (without active error correction) the Gdańsk school claims that is not so for toric / surface codes in 2D, but only in 4D:

Alicki, Fannes & Horodecki, *J Phys A: Math Theor.* **40**:6451 (2007)
Alicki, Fannes & Horodecki, *J Phys A: Math Theor.* **42**:065303 (2009)
Alicki, Horodecki, Horodecki & Horodecki, *Open Systems Inf Dyn* **17**:1 (2010)

KTH/CSC

One is allowed to try something else

Aalto University
School of Science
and Technology

Shouldn’t high bath temperature somehow be a worst-case scenario?

- Feynman-Vernon theory and path integral for spins
- Simple model that gives an AKN-like estimate
- Estimating the errors of computation using the toric code when the physical spins are linearly coupled to a bath of harmonic oscillators

KTH/CSC

Feynman-Vernon, 1963

Aalto University
School of Science and Technology

$$\rho_i^{TOT} \xrightarrow{\rho_i^{TOT} = \rho_i \otimes \rho_{eq}^{bath}} \rho_f^{TOT}$$

$$|i\rangle\langle i| = \rho_i \longrightarrow \rho_f = \mathrm{Tr}_B\left[\rho_f^{TOT}\right] \longrightarrow |f\rangle\langle f|$$

$$P_{if} = \int \psi_i(x_i)\psi_i^*(y_i)\psi_f^*(x_f)\psi_f(y_f)K_{FV}(x_i, y_i, x_f, y_f)\,dx_i dy_i dx_f dy_f$$

$$K_{FV} = \int dq_i dq'_i dq_f \mathcal{D}q\mathcal{D}q'\,\mathcal{D}x\mathcal{D}y\, e^{\frac{i}{\hbar}S_S[x]-\frac{i}{\hbar}S_S[y]+\frac{i}{\hbar}S_I[x,q]-\frac{i}{\hbar}S_I[y,q']+\frac{i}{\hbar}S_B[q]-\frac{i}{\hbar}S_B[q']}\rho_{eq}^{bath}$$

$$K_{FV} = \int \mathcal{D}x\mathcal{D}y\, e^{\frac{i}{\hbar}S_S[x]-\frac{i}{\hbar}S_S[y]+i\Phi[x,y]}$$

Integrate out the bath…assume it is harmonic oscillators and linear coupling…

$$i\Phi[x,y] = \frac{i}{\hbar}S_i[x,y] - \frac{1}{\hbar}S_r[x,y] = \frac{i}{\hbar}\iint_{u\le s}(x_s - y_s)(x_u + y_u)k_i(s-u) - \frac{1}{\hbar}\iint_{u\le s}(x_s - y_s)(x_u - y_u)k_r(s-u)$$

A very explicit final result…

$$k_i(s-u) = \sum_i \frac{C_i^2}{2m_i\omega_i}\sin\omega(s-u)$$

$$k_r(s-u) = \sum_i \frac{C_i^2}{2m_i\omega_i}\coth\frac{\omega_i\hbar\beta}{2}\cos\omega(s-u)$$

KTH/CSC

Path integrals for quantum annealing

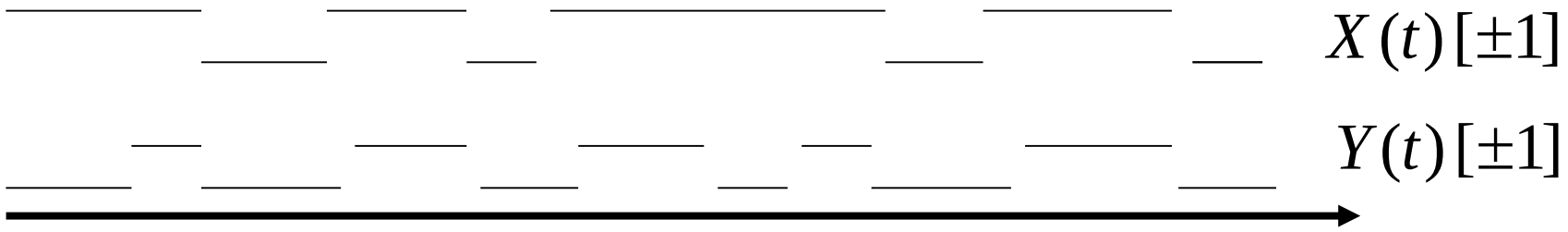
Aalto University
School of Science and Technology

$$H = \sum_{\text{gates } g} c_g \hat{S}^z_{i_{g,1}} \cdots \hat{S}^z_{i_{g,k_g}} + \Gamma \sum_{\text{spins } i} \hat{S}^x_i + \sum_{\substack{\text{spins } i \\ \text{bath } b}} C_{ib} x_b \hat{S}^z_i + H_B$$

Leggett, Chakravarty, Dorsey, Fisher, Garg, Zwerger, *Rev Mod Phys* **59**, 1 (1987)

Bapst, Foini, Krzakala, Semerjian, Zamponi, *Physics Reports*, **523**, 127-205 (2013)

Combining annealing (Bapst *et al*) and baths (Leggett *et al*) gives a multi-spin-boson model of quantum annealing where each spin interacts with its own bath. Spin histories are represented by piece-wise constant paths, each spin having its own influence functional



KTH/CSC

Aalto University
School of Science
and Technology

AKN-like estimate for one-spin-one-bath model

To shorten the notation, write $P_{if}^{(0)} = \langle 1 \rangle_i^f$, $P_{if} = \left\langle e^{i\Phi_{FV}} \right\rangle_i^f$

Assume that the bath is weakly coupled to the system. Then

$$\mathrm{TVD} = \sum_f \left| P_{if} - P_{if}^{(0)} \right| \approx \sum_f \left| \langle \Phi_{FV} \rangle_i^f \right|$$

Integrate out each bath for each spin separately, and use that influence functionals from independent auxiliary systems ***add.***

$$i\Phi_i[X_i, Y_i] = \frac{i}{\hbar} \iint_{u \le s} (X_i - Y_i)(X_i + Y_i) k_i(s-u) - \frac{1}{\hbar} \iint_{u \le s} (X_i - Y_i)(X_i - Y_i) k_r(s-u)$$

$$\Phi_{FV} = \sum_i \Phi_i \approx \eta \cdot N \cdot t$$

$$\mathrm{TVD} \approx \eta \cdot N \cdot t$$

KTH/CSC

Path integrals for more general Hamiltonians

Aalto University
School of Science
and Technology

Built on a coherent state representation:

$$\left(\hat{S}^x, \hat{S}^y, \hat{S}^z\right) \to \vec{S} = \left(\tfrac{1}{2}\sin\theta\cos\phi, \tfrac{1}{2}\sin\theta\sin\phi, \tfrac{1}{2}\cos\theta\right) \quad \hat{H}\left(\hat{S}^x, \hat{S}^y, \hat{S}^z\right) \to H\left(\vec{S}\right)$$

$$\sum_{\text{paths}} \to N_\varepsilon \int \mathcal{D}\theta\mathcal{D}\phi\, e^{i\varepsilon\int \dot{\theta}^2 + (\sin^2\theta)\dot{\phi}^2 + i\int \frac{1}{2}\cos\theta\dot{\phi} - \frac{i}{\hbar}\int H\left(\vec{S}\right)}$$

J. R. Klauder, *Phys. Rev. D* **19**, 2349 (1979)
M. Stone, *Nucl Phys B* **314**, 577 (1989)
Atland & Simons, *Condensed Matter Field Theory* (2006)

The only change (here) is that in the Feynman-Vernon functional the piece-wise constant spin history is replaced by the continuous path (θ,ϕ), and *z*-component of spin changes to cos(θ).

The set-up

KTH/CSC

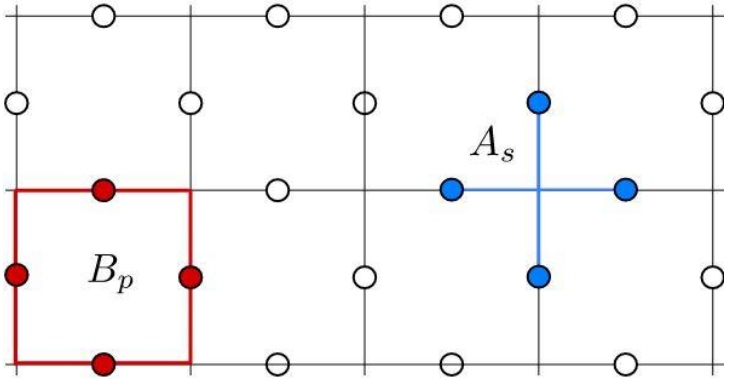
Aalto University
School of Science
and Technology

logical qubits

physical spins

$$H = \sum_{\text{gates } g} c_g \hat{\sigma}_{i_{g,1}}^{\alpha_{g,1}} \cdots \hat{\sigma}_{i_{g,k_g}}^{\alpha_{g,k_g}} + \sum_{\text{bath } b} C_{bi} x_b \left(\sum_{\text{spins } i} \hat{S}_i^z \right) + H_B$$

$$A_s = \prod_{j \in \text{star}(s)} \hat{S}_j^x \quad B_p = \prod_{j \in \partial p} \hat{S}_j^z$$



Problems: (*i*) the logical qubits and the physical spins are so different; (*ii*) the stabilizers are measured continuously.

source: OpenCourseWare *TUDelft*

The stabilizers all commute. The state after measuring is hence a density matrix on the eigenspace of dimension 2^k (4 for toric codes).

Assume that one can (somehow) deal with what happens when the measured values of the stabilizers change, and that happens rarely. Then all that needs to be analyzed is dynamics at *constant* stabilizers.

Basis change

KTH/CSC

Aalto University
School of Science
and Technology

Eigenfunctions:

$$\underbrace{|\sigma_1,\ldots,\sigma_k}_{\text{logical qubits}};\underbrace{s_1,\ldots,s_{N-k}\rangle}_{\text{stabilizers}} = \sum_{i_1,\ldots,i_N} A_{i_1,\ldots,i_N}^{\sigma_1,\ldots,\sigma_k;s_1,\ldots,s_{N-k}} \underbrace{|i_1,\ldots,i_N\rangle}_{\text{physical qubits}}$$

The interaction of the physical spins ***i*** with one bath oscillator is then a complicated interaction Hamiltonian for the logical qubits $\boldsymbol{\sigma}$.

$$\langle \boldsymbol{\sigma}';\mathbf{s}|C_{bi}x_b\sum_{\text{spins } i}\hat{S}_i^z|\boldsymbol{\sigma};\mathbf{s}\rangle = x_b\sum_{\mathbf{i}}C_{bi}\left(A_{\mathbf{i}}^{\boldsymbol{\sigma}',\mathbf{s}}\right)^* A_{\mathbf{i}}^{\boldsymbol{\sigma},\mathbf{s}}\left(\sum_{s=1}^{N}(-1)^{i_s}\right) \equiv x_b Q(\boldsymbol{\sigma}',\boldsymbol{\sigma};\mathbf{s})$$

The expansion coefficients A for the toric code are known. At least for the ground state where they are all $N^{-\frac{1}{2}}$ or zero. It is also known that the matrix elements between the ground states of a local operator (such as $\hat{S}_i^z$) are exponentially small.

Estimates

KTH/CSC

Aalto University
School of Science
and Technology

Integrating out the bath gives

$$i\Phi_{FV}\left[\{\theta,\phi\}^F,\{\theta,\phi\}^B\right]=\frac{i}{\hbar}\iint_{u\le s}(Q^F-Q^B)(Q^F+Q^B)k_i-\frac{1}{\hbar}\iint_{u\le s}(Q^F-Q^B)(Q^F-Q^B)k_r$$

where the interactions with the bath are described by:

$$Q^F\left(\boldsymbol{\theta}^F,\boldsymbol{\varphi}^F;\mathbf{s}\right)=\sum_{\boldsymbol{\sigma}',\boldsymbol{\sigma}}\left\langle\boldsymbol{\theta}^F,\boldsymbol{\varphi}^F\middle|\boldsymbol{\sigma}'\right\rangle Q\left(\boldsymbol{\sigma}',\boldsymbol{\sigma};\mathbf{s}\right)\left\langle\boldsymbol{\sigma}\middle|\boldsymbol{\theta}^F,\boldsymbol{\varphi}^F\right\rangle$$

$$Q^B\left(\boldsymbol{\theta}^B,\boldsymbol{\varphi}^B;\mathbf{s}\right)=\sum_{\boldsymbol{\sigma}',\boldsymbol{\sigma}}\left\langle\boldsymbol{\theta}^B,\boldsymbol{\varphi}^B\middle|\boldsymbol{\sigma}'\right\rangle Q\left(\boldsymbol{\sigma}',\boldsymbol{\sigma};\mathbf{s}\right)\left\langle\boldsymbol{\sigma}\middle|\boldsymbol{\theta}^B,\boldsymbol{\varphi}^B\right\rangle$$

$\left\langle\boldsymbol{\sigma}',\mathbf{s}\middle|\boldsymbol{\sigma},\mathbf{s}\right\rangle=\sum_{\mathbf{i}}\left(A_{\mathbf{i}}^{\boldsymbol{\sigma}',\mathbf{s}}\right)^*A_{\mathbf{i}}^{\boldsymbol{\sigma},\mathbf{s}}=\mathbf{1}_{\boldsymbol{\sigma}',\boldsymbol{\sigma}}$ so reasonable if $A_{\mathbf{i}}^{\boldsymbol{\sigma},\mathbf{s}}\approx 2^{-\frac{N}{2}}$ and $Q\left(\boldsymbol{\sigma}',\boldsymbol{\sigma},\mathbf{s}\right)\approx 2^{-\frac{N}{2}}$

$\left\langle\theta,\phi\middle|\uparrow\right\rangle=e^{i\frac{\phi}{2}}\cos\frac{\theta}{2}$, $\left\langle\theta,\phi\middle|\uparrow\right\rangle=e^{i\frac{\phi}{2}}\cos\frac{\theta}{2}$ hence $\left\langle\left|Q^{\mathrm{F}}\right|^2\right\rangle\approx\left\langle\left|Q^B\right|^2\right\rangle\approx 2^{2k}\cdot 2^{-k}\cdot 2^{-N}$

Surface codes, at least in the ground state, are *protected* against errors from interaction with a spin-boson bath

Conclusions

Quantum computing and quantum error correction is an enormous field – the more one looks, the more one finds.

The important example of toric codes / surface codes seem mainly to have been looked at using a limited (random external field, Markov evolution) set of error models

It has been argued that this is a problem (Alicki, Kalai,…)

The Feynman-Vernon method allows to look at error models describing an interacting with a bath of harmonic oscillators.

At least in that case there does not seem to be a problem.

Open (hard) problems

KTH/CSC

Aalto University
School of Science
and Technology

One would like to have more numbers. How “high” is “high temperature”? How good is the assumption of Markovian evolution actually?

Coupling to a heat bath of oscillators lead to very pretty formulae but are perhaps not so realistic as the main error mechanisms in real materials.

The excitations in a real material at very low temperature are said to be mainly localized (nuclear spins, impurities), and their influence would be much harder to evaluate, see *e.g.*

N V Prokof'ev & P C E Stamp, “Theory of the spin bath” *Reports on Progress in Physics* **63**:669 (2000)

KTH/CSC

Aalto University
School of Science
and Technology

Thanks

Karol Życzkowski

Sergey Pershoguba

Gil Kalai

Petteri Kaski

Vetenskapsrådet