Department of Mathematics and Statistics
Stokastiset prosessit
Exercise 2
30.05.2008

1. Represent an event $\{T_A = n\}$ by using the history of the chain given the chain starts from $X_0 = i$. Represent the probability of the event $\{T_A = n\}$ by using the transition probabilities $p_{ij}^{(m)}$. [Have a look at the formula (2.11) in lecture notes.]

2. Let (X_n) be a Markov chain with state space $S = \{1, 2, 3, 4, 5\}$ and transition probability matrix

$$P = \begin{pmatrix} & & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ & \frac{2}{3} & & \frac{1}{3} & \\ & & 1 & & \\ & \frac{7}{8} & & \frac{1}{8} & \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & & \end{pmatrix}$$

a) Draw the corresponding state transition diagram

b) Find all the absorption sets

c) Determine the absorption probabilities to from every starting state $i = 1, \ldots, 5$ to every absorption set A.

3. Consider the money game example 2.1 in lecture notes. Suppose the player A starts with i euros and player B starts with $d - i$ euros. Calculate the probability for the event "player A loses the game". Draw the losing probability as a function of starting money i when $p = q = \frac{1}{2}$ and when $p = \frac{3}{4}$ ja $q = \frac{1}{4}$. [Hint. Apply suitable example in the lecture notes and use the knowledge $h_d = 0$.]

4. Show that for every time instance $n, \widehat{n}$ and m

$$\begin{aligned} &\mathbf{P}\,(\, X_{n+m} = i_{n+m}, \ldots, X_{n+1} = i_{n+1} \,|\, X_n = i_n, \ldots, X_0 = i_0 \,) \\ &= \mathbf{P}\,(\, X_{\widehat{n}+m} = i_{n+m}, \ldots, X_{\widehat{n}+1} = i_{n+1} \,|\, X_{\widehat{n}} = i_n \,)\,. \end{aligned}$$

[Hint. Use the path probability formula]

5. Consider a birth-death chain (SK-ketju in notes) (X_n) on $\mathbb{N}$. Let $T_i = \min\{\, n \in \mathbb{N} : \; X_n = i \,\}$ (Note. if $i \neq 0$, then T_i is the first hitting time, but not an absorption time). Show that

$$\mathbf{P}_2\,(\, T_0 = n \,) = \sum_{0<m<n} \mathbf{P}_2\,(\, T_1 = m \,)\, \mathbf{P}_1\,(\, T_0 = n - m \,)\,.$$

Let $G_i(t) := \mathbf{E}_i\, t^{\,T_0}$ and $H_{ij}(t) := \mathbf{E}_i\, t^{\,T_j}$. Use the previous identity to show that

$$G_2(t) = H_{21}(t) G_1(t).$$