On a class of elliptic problems with critical growth in the gradient

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Abstract

This talk focuses on the existence and multiplicity of solutions for a boundary value problem of the form

\[-\Delta u = c(x)u + \mu(x)|\nabla u|^2 + h(x), \quad \text{in } \Omega,\]

\[u = 0, \quad \text{on } \partial \Omega.\]

Solutions are searched in the function space \(H^1_0(\Omega) \cap L^\infty(\Omega)\) where \(\Omega \subset \mathbb{R}^N, N \geq 2\), is a bounded domain with smooth boundary. It is assumed that \(c, h\) belong to \(L^p(\Omega)\) for some \(p > N/2\) and \(\mu\) belongs to \(L^\infty(\Omega)\).

In the case where \(c(x) \leq \alpha_0 < 0\), now referred to as the coercive case, this problem has been studied since the 80’s and the existence of a unique solution is the rule. Recently, other cases (in particular assuming that \(c(x) \geq 0\) or that \(c(x)\) changes sign) started to be considered. We shall present some of the main contributions in these non-coercive cases. We will see that both existence and uniqueness may now be lost.

The talk is based in joint works with Colette De Coster (Université Polytechnique des Hauts-de-France, Valenciennes (France)) and Louis Jeanjean (Université de Franche-Comté, Besançon (France)).