

Diff. int. II

20.1.2004

1. Formulate and prove the Cauchy-Schwarz inequality in \mathbb{R}^n .

2. Show that the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$,

$$f(x) = \begin{cases} \frac{x_1^3}{|x|} & , x = (x_1, x_2) \neq 0, \\ 0 & , x = 0, \end{cases}$$

has a derivative at 0.

3. Find the value of the integral $\iint_D x \, dx \, dy$ where D is the set bounded by x -axis and the lines $x=1$ and $y=x$.

4. Find

$$\iint_D \frac{dx \, dy}{x \, y}$$

where D is the bounded set between the graphs of the functions $y=x$ and $y=x^2$.

5. Find the maximum and minimum points of the function $f(x, y) = 2xy$ in the closed disk $\bar{B}(0, 2)$.

Department of Mathematics and Statistics
Vector analysis
Final exam
26.1.2006

1. Find a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ such that $\partial_1 f(0)$ and $\partial_2 f(0)$ exist but f is not continuous at $(0,0)$.
2. Does the function $f : \mathbb{R}^2 \setminus \{(0,0)\} \rightarrow \mathbb{R}$

$$f(x, y) = \frac{2x^2y}{x^4 + y^2}$$

have a limit at $(0,0)$?

3. Find

$$\iint_D y \, dx \, dy,$$

when

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, x^2 \leq y \leq x\}.$$

4. Give the definitions for the integrals

$$\int_{\gamma} f \, ds \quad \text{and} \quad \int_{\gamma} F \cdot d\bar{s}$$

where $\gamma : [a, b] \rightarrow \mathbb{R}^2$ is C^1 -path. Compute the integrals for $\gamma(t) = (t, 1)$, $t \in [0, 2]$, and $f(x) = |x|^2$, $F(x) = x$.

5. Find the local extremum points if the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4.$$

Are the points local minimums or maximums?