Department of Mathematics and Statistics
Differential and integral calculus II
Midterm exam 1
1.11.2004

1. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$,
$$f(x, y) = (xy, -x, y),$$
and $g : \mathbb{R}^3 \to \mathbb{R}$,
$$g(x, y, z) = x - ye^z.$$
Calculate $\nabla(g \circ f)(-1, 0)$.

2. Find vectors v to whose direction the directional derivative of a function $f : \mathbb{R}^2 \to \mathbb{R}$,
$$f(x, y) = x^2 + 2xy - \sqrt{3}y^2,$$
at the point $(0, 1)$ is equal to zero.

3. Find the biggest and the smallest value of a function
$$f(x, y) = x^3 - 3xy^2$$
in the set $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leqslant 4\}$.

4. Find the set $D \subset \mathbb{R}^3$ in which the restriction of a mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$,
$$f(x, y, z) = (x, y^2, z^4),$$
is locally invertible. Is the restriction of f to the set D globally invertible, that is, is $f|_D : D \to \mathbb{R}^3$ injection?

Department of Mathematics and Statistics
Vector Analysis
1. examination
17.10.2005

1. Find if the function $f : \mathbb{R} \setminus \{(0,0)\} \to \mathbb{R}$,

$$f(x,y) = \frac{x^2}{x^2+y^2},$$

has a limit at $(0,0)$?

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = y \sin x + y$. Find $\partial_1 f(x,y)$, $\partial_2 f(x,y)$ and $\partial_1 \partial_2 f(x,y)$.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$,

$$\begin{aligned} f(x,y) &= \frac{x^2 y}{|x|+|y|}, && (x,y) \neq (0,0), \\ &= 0, && (x,y) = (0,0). \end{aligned}$$

Is f continuous at $(0,0)$?

4. Let $f, g : \mathbb{R}^2 \to \mathbb{R}^2$,

$$f(x,y) = (xy, 1), \quad g(x,y) = (y, x+1).$$

Find the formula for the mapping $g \circ f$. Find the matrix of the derivative $(g \circ f)'(x,y)$.