Department of Mathematics and Statistics
Topology II
Final exam
18.12.2007

1. Prove that the sets $[a, b[\times [c, d[$ $(a, b, c, d \in \mathbb{R},\ a < b,\ c < d)$ form a basis for a topology $\mathcal{T}$ of $\mathbb{R}^2$. Are two straight lines in $\mathbb{R}^2$ always homeomorphic with topologies induced by $\mathcal{T}$?

2. Let $(X, \mathcal{T})$ be a topological space and let $A \subset X$. Prove that the following conditions are equivalent:
(1) A is dense in X
(2) $A \cap U \neq \emptyset \quad \forall U \in \mathcal{T} \setminus \{\emptyset\}$
(3) int $(X \setminus A) = \emptyset$ (int = interior).

3. (a) Give an example of an arcwise connected subspace of $\mathbb{R}^2$ with a non–arcwise connected closure.
(b) Prove that the image of a connected space under a continuous mapping is connected.

4. Let X and Y be Hausdorff–spaces, $f : X \to Y$ a continuous mapping and $(K_n)_{n=1}^{\infty}$ a decreasing sequence of compact subsets of X. Prove that $f(\bigcap\limits_{n=1}^{\infty} K_n) = \bigcap\limits_{n=1}^{\infty} f(K_n)$.

5. A space X is totally disconnected if its components are singletons. Prove that the product space $\prod\limits_{i \in I} X_i$ of totally disconnected spaces X_i $(i \in I)$ is totally disconnected.