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Department of Mathematics and Statistics
Topology I
Midterm examination 2, 6. 5. 2008

1. Consider the set

$$A = \{ (x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 7,\ y \geq 1 \} \subset \mathbb{R}^2.$$

Determine the interior $\operatorname{int} A$, the boundary ∂A, the closure $\overline{A}$ and the exterior $\operatorname{ext} A$ of A in $\mathbb{R}^2$.

2. Find an example of two homeomorphic metric spaces one of which is bounded and the other is unbounded.

3. a) Show that the mapping $f\colon [\frac{3}{2}, 2] \to [\frac{3}{2}, 2]$,

$$f(x) = \frac{1}{2}\left(x + \frac{3}{x}\right),$$

is a contraction.

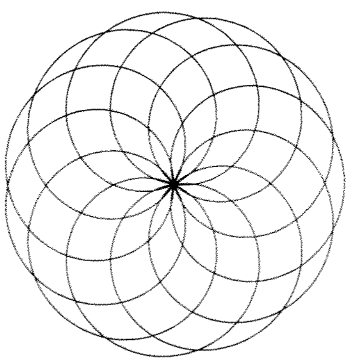
b) Explain why $[\frac{3}{2}, 2]$ is complete.

c) Consider the sequence $(x_n)_{n \in \mathbb{N}}$ for which $x_0 = 2$ and $x_{n+1} = f(x_n)$ for all $n \in \mathbb{N} = \{0, 1, 2, \ldots\}$. Show that $a = \lim_{n \to \infty} x_n$ exists and determine the value a.

4. Put $f\colon [0, 2\pi] \to \mathbb{R}^2$,

$$f(t) = (\cos t + \cos 12t, \sin t + \sin 12t).$$

The range of f, i.e., the set $A = f[0, 2\pi]$ is illustrated in the figure below. Determine if A is a) compact, b) connected, c) open in the plane $\mathbb{R}^2$?



[You may state the needed properties of f without proof. The plane is known to be connected.]