HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

Department of Mathematics and Statistics
Topology I
Midterm examination 1, 26. 2. 2008

1. Let E be an inner product space. In order to be able to reconstruct the space one has to know the vector space structure and one of the following: the inner product, the norm or the metric. If one of these is known, the other two can be determined. Explain how this can be done. (The explanation should consist of 6 formulas and short motivations.)

2. Consider

$$A = \{ (x, y) \in \mathbb{R}^2 \mid x > 0,\ x^2 + y^2 \geq 1 \}.$$

Determine if A is an open subset of the plane $\mathbb{R}^2$.

3. a) Let E be a nontrivial normed space, i.e., $E \neq \{\overline{0}\}$, and let $r \geq 0$. Show that there are points a and b whose distance is r.
b) Give an example of a metric space (X, d) whose metric cannot be a metric of any normed space.

4. Show that the mapping $f\colon \mathbb{R}^3 \to \mathbb{R}$,

$$f(x, y, z) = x + e^y z,$$

is continuous, where the continuity of $\exp\colon \mathbb{R} \to \mathbb{R}$, $\exp(x) = e^x$, is assumed. Is it true that f is a Lipschitz-mapping?