Topology I

Mid-Term Test I

October,21,2005 starts at 10:00 in C124
duration: 2 hours

1. (20 marks) Let (X,τ) be a topological space and let $A \subseteq X$ be an open set. Prove that $A \cap \overline{B} \subseteq \overline{A \cap B}$, for any $B \subseteq X$.

2. (20 marks) Let X be an infinite cofinite space. Show that every infinite subset of X is dense.

3. (20 marks) Let (X,τ) be a topological space and let $A \subseteq X$. Show that A is closed if and only if $b(A) \subset A$.

4. (20 marks) Let τ be the class of subsets of $\mathbb{N}$ consisting of $\varnothing$ and all subsets of $\mathbb{N}$ of the form $E_n = \{n, n+1, n+2, \ldots\}$ with $n \in \mathbb{N}$. Show that τ is a topology on $\mathbb{N}$.

5. (20 marks) Let (X,τ) be an indiscrete space and let A be a nonempty proper subset of X. Find $\overline{A}$.

Topology 1
mid-term
Fall 2006
Kennedy

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1. Prove that the reals are uncountable.

2. Prove that the set of all subsets of the natural numbers has the same cardinality as the reals.

3. Prove that any open set in a metric space is a union of open spheres and conversely.

4. Prove that any open set in the real line is the union of a countable disjoint class of open intervals. (This is worth 2 questions.)

5. Extra credit: Prove that a complete metric space cannot be written as the countable union of nowhere dense sets.