STATIONARY TIME SERIES, 8–10 CREDIT POINTS (intermediate or master level). Literature: James Hamilton's Time Series Analysis, Chapters 1–5 (and 6). Examiner: University lecturer Pekka Pere.

General examination 24.1.2008

Answer three of the first four questions for completion of 8 study points (intermediate level). To achieve 10 study points you should answer also the fourth question (master level). Please mark clearly which questions you have answered! Each of these questions is worth six points. Please return the questions. Thank you!

1. Hall and Saidi (2007)[3] estimated the models below for Canadian (real) gross national product ($\Delta y_{1t} = y_{1t} - y_{1t-1}$) and (nominal short) interest (y_{2t}):

$$\Delta y_{1t} = 2552,456 + 0,345\Delta y_{1t-1} + \hat{\varepsilon}_{1t}, \quad \hat{\sigma}_1 = 3545,353$$

$$y_{2t} = 1,298 + 0,872 y_{2t-1} + \hat{\varepsilon}_{2t} - 0,307\hat{\varepsilon}_{2t-1}, \quad \hat{\sigma}_2 = 1,273.$$

Above $\varepsilon_t \sim \mathsf{IID}(0,\sigma_i^2)$, "^" indicates an estimated quantity and $\hat{\sigma}_i$ is the standard error of the respective estimated innovation ($i = 1,2$). The observations are quaterly and cover the years 1970–1989 (80 observations). The models pass diagnostic tests. Let us assume the happy circumstance that the estimated models match perfectly with the underlying processes.

a) Are processes Δy_{1t} and y_{2t} weakly stationary? Explain.

b) Calculate the unconditional means $\mathsf{E}(\Delta y_{1t})$ and $\mathsf{E}(y_{2t})$. Are the means constant over time? Explain.

c) Explain carefully how the autocorrelation functions of the processes behave (it is not necessary to write down the detailed formula for the latter process). Calculate the first four autocorrelations of Δy_{1t}. You gain an extra point if you can calculate the first four autocorrelations of y_{2t}.

2. Let us investigate forecasting of the processes above. Let the last observations be $\Delta y_{1T} = 4000$ and $y_{2T} = 10$. It is easy to calculate the value ε_{2T} takes because the parameter values are known. Let ε_{2T} equal 10. Calculate the forecasts or the conditional means $\mathsf{E}(\Delta y_{1t+1}|\Delta y_{1t}, \Delta y_{1t-1}, \ldots)$, $\mathsf{E}(\Delta y_{1t+2}|\Delta y_{1t}, \Delta y_{1t-1}, \ldots)$, $\mathsf{E}(y_{2t+1}|y_{2t}, y_{2t-1}, \ldots)$ and $\mathsf{E}(y_{2t+2}|y_{2t}, y_{2t-1}, \ldots)$. (Hint1: Wiener–Kolmogorov formula. Hint2: $\hat{y}_{t+s|t} - \mu = \phi(\hat{y}_{t+s-1|t} - \mu)$, if $s = 2, 3, \ldots$ Hint3: $\phi + \theta = \phi(1 + \theta L) + \theta(1 - \phi L)$.) Do the forecasts converge as the forecast horizon s tends to infinity? If yes then to what do the forecasts converge to?

[3] Optimal Tests of Noncorrelation Between Multivariate Time Series. *J. American Statistical Association*, 102. 938 951.

3.

a) It is written in a thesis: "AR and MA parameters of an ARMA(p,q) process must not take equal values because equal values would cause problems in the estimation of the parameter values (e.g. Hamilton 1994, 60–61)." Is the statement correct? Explain.

b) Present the Wiener–Kolmogorov formula and explain what it is used for.

c) Your task is to fit an ARMA(p, q) model to an empirical time series. Explain how the sample autocorrelation and partial autocorrelation functions can be used to determine the indeces p and q. Explain carefully the associated distribution theory.

4. Let the initial value y_0 of the process $\{y_t\}_{t=0}^{\infty}$ be 1 with probability 1/2 and -1 with probability 1/2. The ensuing values of y_t are determined by the formula

$$y_t = (-1)^t y_0, \quad t = 1, 2, \ldots.$$

a) Derive the expected value, variance and autocovariance function of the process.

b) Is y_t a weakly stationary process? Explain. Does the autocorrelation function of a weakly stationary process always decay for large lags?

c) Derive from the results of the general theory of forecasting the optimal (mean square error minimising) linear forecast for y_{t+1} when the conditioning information is y_t only (no constant). (If you do not remember the general result by heart, you can of course derive the general result as well.) What is the mean square error of the forecast?

5. Let a process be weakly stationary and the associated autocovariances (γ_j) absolutely summable.

a) Define the population spectrum. Explain it in words.

b) Moreover, let the process be *i)* MA(∞) or *ii)* ARMA(p,q). Give and explain the population spectrums for these processes.

c) Define and explain what are population spectrums like if the process is *i)* white noise *ii)* MA(1) with MA coefficient $\theta > 0$ or $\theta < 0$ and *iii)* AR(1) with AR coefficient $\phi > 0$ or $\phi < 0$. Explain the spectrums in words.

Auxiliary result:

$$\begin{aligned} \exp(i\omega) &= \cos(\omega) + i\sin(\omega), \\ \exp(-i\omega) &= \cos(\omega) - i\sin(\omega). \end{aligned}$$