STATIONARY TIME SERIES, 8–10 CREDIT POINTS. Literature: James Hamilton's Time Series Analysis, Chapters 1–5 (and 6). Examiner: University lecturer Pekka Pere.

1st intermediate examination 29.10.2007

Please answer three of the four questions and mark clearly which questions are to be inspected. Each question is worth six points. Please return the questions. Thank you!

1. Let the variable y_t (not necessarily random) follow a pth order difference equation

$$y_t = \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + w_t$$

(with obvious notation).

a) When is the solution of the equation stabile?

a) What is the dynamic multiplier? Explain its meaning in words.

b) Let us assume that a permanent change in the value of w_t takes place. How does the long run solution of y_t change and how do the dynamic multipliers relate to the answer?

2. Let$\{\varepsilon_t\}_{t=-\infty}^{\infty}$ be white or $\mathsf{E}(\varepsilon_t) = 0$, $\mathsf{E}(\varepsilon_t^2) = \sigma^2 > 0$ and $\mathsf{E}(\varepsilon_t \varepsilon_{t-j}) = 0$, if $j \neq 0$. Let us create new processes $y_t = (-1)^t \varepsilon_t$ and $z_t = y_t + \varepsilon_t$, where $t = 0, \pm 1, \pm 2, \ldots$

a) Is y_t weakly stationary? Is it white noise?

b) Is z_t weakly stationary? Is the sum of two weakly stationary processes always weakly stationary? Explain.

3. Let y_t follow the AR(2)-process

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t,$$

where $\phi_1 = 1/2$, $\phi_2 = 3/16$, ε_t is white noise ($\mathsf{E}(\varepsilon_t) = 0$, $\mathsf{E}(\varepsilon_t^2) = \sigma^2 > 0$ and $\mathsf{E}(\varepsilon_t \varepsilon_{t-j}) = 0$, if $j \neq 0$) and $t = 0, \pm 1, \pm 2, \ldots$

a) What is the characteristic equation associated with the process?

b) Is the process weakly stationary? Explain.

c) Prove that the factorisation $1 - \phi_1 L - \phi_2 L^2 = (1 - \lambda_1 L)(1 - \lambda_2 L)$ holds with the given parameter values. The quantaties λ_1 and λ_2 are the roots of the characteristic equation.

d) Give a recursive formula for the autocorrelations ρ_j of the process. Prove with the formula that $\rho_1 = \phi_1/(1 - \phi_2)$.

e) Calculate numerical values for the autocorrelations at lags 1, 2 and 3.

f) Describe the behaviour of the autocorrelation function for these and larger lags. How is the behaviour affected by the roots λ_1 and λ_2 being complex valued or not?

4. Let $\{y_t\}_{t=-\infty}^{\infty}$ follow an MA(1) process

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1},$$

where ε_t is white noise($\mathsf{E}(\varepsilon_t) = 0$, $\mathsf{E}(\varepsilon_t^2) = \sigma^2 > 0$ and $\mathsf{E}(\varepsilon_t\varepsilon_{t-j}) = 0$, if $j \neq 0$).

a) Does the magnitude of the parameter θ affect potential stationarity of the process, and if it does then how? Explain.

b) Derive the mean, variance and autocorrelationfunction of the process.

c) What is meant by invertibility of an MA process? When is an MA(1) process invertible?

Auxiliary result: The roots of the second order polynomial $ax^2 + bx + c = 0$ ($a \neq 0$) are

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$