

Tämä kuulustelu kohdistuu osoitteessa <http://www.math.helsinki.fi/~okanerva/sim02/luentopaivakirja.html> mainittuihin Law'n-Keltonin kirjan¹ kohtiin.

Vastaukset saa antaa suomeksi.

1. Suppose that X and Y are jointly discrete random variables with

$$p(x, y) = \begin{cases} \frac{x+y}{30} & \text{for } x = 0, 1, 2 \text{ and } y = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases} .$$

- (a) Compute and plot $p_X(x)$ and $p_Y(y)$.
 - (b) Are X and Y independent?
 - (c) Compute and plot $F_X(x)$ and $F_Y(y)$.
 - (d) Compute $E(X)$, $\text{Var}(X)$, $E(Y)$, $\text{Var}(Y)$, $\text{Cov}(X, Y)$ and $\text{Cor}(X, Y)$.
2. Explain the concepts of *verification*, *validation*, and *credibility* of a simulation model. Briefly describe several techniques related to each of the three.
3. (a) Describe the pros and cons of the following three methods of choosing an input probability distribution (based on collected data): *trace-driven simulation*, *empirical distribution*, fitting a *theoretical distribution*.
- (b) What is meant by calling the (stochastic) arrival process $\{N(t), t \geq 0\}$ a *Poisson process*? What can be said about the interarrival times A_1, A_2, A_3, \dots for a Poisson process?
4. (a) Consider the (Fibonacci) generator

$$Z_i = (Z_{i-1} + Z_{i-2}) \pmod{m} .$$

- i. Show that this generator can never produce the following arrangement of three consecutive output values:

$$U_{i-2} < U_i < U_{i-1} .$$

- ii. "Show" that the arrangement in i should occur with probability $\frac{1}{6}$ for a "perfect" random-number generator. What is your conclusion about the Fibonacci generator?
- (b) Describe the *acceptance-rejection* method for generating random variates **or** how to generate the times of arrival t_1, t_2, t_3, \dots for a Poisson process with rate $\lambda > 0$.
5. Describe the main points of the statistical analysis for terminating simulations. Especially consider the significance of independence or dependence — and perhaps some pitfalls of simulation studies in general.

¹*Simulation modeling and analysis* (McGraw-Hill, 2000; kolmas painos)