

1. Suppose $T_n, n=1, 2, \dots$, is a sequence of estimators for the parameter θ , for which

$$\sqrt{n}(T_n - \theta) \xrightarrow{w} N(0, \sigma^2) \quad (n \rightarrow \infty).$$

To estimate $g(\theta)$ we use $g(T_n)$ as an estimator. Derive the asymptotic probability distribution of the sequence

$g(T_n), n=1, 2, \dots$, of estimators

in case $g'(\theta) = 0, g''(\theta) \neq 0$

[Hint: Taylor-series]

2. Show that the likelihood equation has a unique solution for distributions belonging to the exponential family.

[1-dimensional parameter, canonical parametrization].

3. The consistency of a sequence of estimators. Present a definition and justify why consistency is an important property ~~in~~ in statistical inference.

Derive a consistent sequence of estimators for the parameter

$$P = P(\bar{X} \leq x_0) \quad (x_0 \text{ fixed})$$

Give justifications for the consistency property.

4. Determine the LR-test statistic in the case $\bar{X} \sim N(\mu, 1)$:

$$H_0: \mu = \mu_0$$

$$H_1: \mu \neq \mu_0.$$

Describe this also graphically by using the logarithmic likelihood function.