

Optimointi 1
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1. A grocery store is open on weekdays from 9.00–20.00. The shopkeeper estimates that the number of cashiers needed is at least 2 from 9.00–11.00, at least 4 from 11.00–13.00, at least 3 from 13.00–16.00, at least 5 from 16.00–19.00 and at least 2 from 19.00–20.00. The work shifts of the cashiers are 9.00–16.00, 11.00–18.00 and 13.00–20.00. How many cashiers should be hired for each shift, if the shopkeeper wants to minimize the amount of wages he has to pay to the cashiers (every cashier has the same wages)? Use the simplex method and give all solutions.
2. Find the dual of the following LP problem:

$$\begin{aligned}\min q &= x_1 + 3x_2 + 2x_3 \\ x_1, x_2, x_3 &\geq 0 \\ x_1 - 2x_2 + x_3 - 4 &\geq 0 \\ 2x_1 - 3x_3 - 2 &\geq 0 \\ -x_1 + x_2 + 3 &\geq 0.\end{aligned}$$

Solve the problem and the dual problem using a composite pivotal scheme.

3. Solve, for all $\kappa \geq 0$, the following problem for parametrizing the cost vector:

$$\begin{aligned}\min q &= (1 + \kappa)x_1 + x_2 - (2 + \kappa)x_3 \\ x_1, x_2, x_3 &\geq 0 \\ -x_1 - x_2 + 3x_3 - 2 &\leq 0 \\ x_1 + x_2 + x_3 + 1 &\geq 0 \\ x_1 - x_2 + x_3 + 1 &\geq 0.\end{aligned}$$

Also give the definition set of the problem.

4. Solve the following problem by means of the simplex method for quadratic programming:

$$\begin{aligned}\min q &= x_1^2 + 2x_2^2 - 2x_1x_2 - 8x_1 + 6x_2 \\ x_1, x_2 &\geq 0 \\ -x_1 + x_2 &\geq 2 \\ 2x_1 + x_2 &\leq 10.\end{aligned}$$