Department of Mathematics, University of Helsinki
Numerical methods and the C language, spring 2008
Exam, February 27, 2008 (2 hours)

Answer four questions! You may use either C++ and matutl04 or C and gmatutl in your work. Header files such as #include <iostream> are not required.

1. (a) Compare the floating point arithmetic with the properties of the real numbers. What types of errors are likely to occur in floating point arithmetic, and how can they be avoided ?

   (b) Describe some mathematical functionalities usually offered by programming languages and environments. What additional numerical properties would you like the C/C++ language to have, and on what basis ?

   (c) How can one examine how sensitive to errors the numerical solution for an $n \times n$ linear system is ? What methods are there for solving such a system ? Use a method of your choice to solve the system $Ax = b$, where

$$A = \begin{bmatrix} 3.0 & 2.0 & 1.0 \\ 4.0 & 6.0 & 5.0 \\ 7.0 & 8.0 & 9.0 \end{bmatrix} \qquad b = \begin{bmatrix} 2.0 \\ -1.0 \\ -2.0 \end{bmatrix}.$$

2. Make a program that generates a random $n \times n$ matrix A, computes its inverse b with matutl04.cpp, and prints the number $\mathrm{norm}(Id - A * b)$ which should be zero. Here Id is the $n \times n$ identity matrix. Use the programs from matutl04.cpp. Writing down the standard header files such as <stdio.h> is not required.

3. Suppose that we have two vectors of the same length x, y. Make a program that prints the value of the scalar product of these vectors.

4. Describe the usage of *one* of the datatypes Mat_DP or gsl_matrix used on the course, especially considering the following:

   (a) Declaration, memory allocation, component access and printing.

   (b) Show how to handle matrices, including summing two matrices, reading a matrix from a file, and the matrix product of two matrices. You may use the programs from matutl04.cpp/gmatutl.c.

   (c) Make a program which demonstrates the usage of the functions above.

5. Consider the problem of fitting a LSQ line with the data points $(x_i, y_i), i = 1, ..., n$. It is assumed that the line goes through a fixed point (x_0, y_0). Thus we want to minimize the function

$$s(c) = \sum_{i=1}^{n} (y_i - y_0 - c(x_i - x_0))^2.$$

   (a) Write the formula for $s'(c)$ and solve the equation $s'(c) = 0$ for c.

(b) We assume that the data is in the file a.dat (in the standard format of the course). Write a program that reads the data and computes the value of c and writes it on the screen.

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Department of Mathematics, University of Helsinki
Numerical methods and the C language, spring 2008
Examination 2, May 6, 2008

Answer four questions! You may use either C++ and matutl02 or C and gmatutl in your work. Header files such as #include <iostream> are not required.

1. We consider a quadrature formula of the form

$$\int_0^3 f(x)dx = c_1 f(0) + c_2 f(1) + c_3 f(3)\,.$$

Determine the coefficients c_1, c_2, c_3 so that the formula holds as equality for $x^j, j = 0, 1, 2$.

2. Write a program that experimentally tests a subprogram that carries out numerical integration.

(a) Give an example of a suitable family of test functions for which the integral can also be exactly computed and print the exact value, numerical value and error of an integral.

(b) Describe carefully the definition of the function to be integrated, the syntax of the integration subprogram, and give an example of a fully functional test program.

(c) Explain what are the possible further refinements of your program and how these could be achieved.

3. (a) Describe the Newton algorithm for solving the system $f_1(x,y) = 0\,, f_2(x,y) = 0\,.$

(b) Consider the nonlinear system

$$x^2 - 2x - y + 0.5 = 0, \quad x^2 + 4y^2 - 4 = 0\,.$$

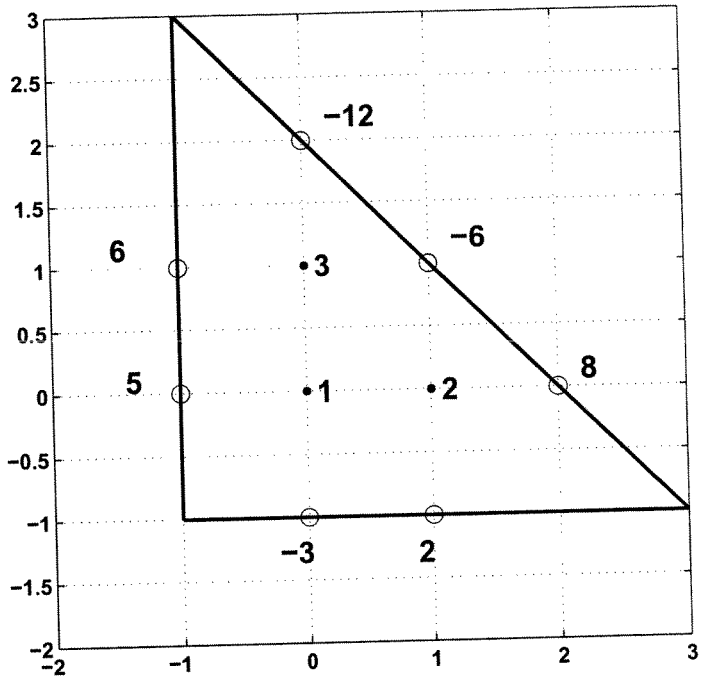
Use Newton's method with the starting value $(x_0, y_0) = (2.00, 0.25)$ and compute manually (x_1, y_1). Comparing the function values at (x_0, y_0) and (x_1, y_1) was this a succesful step?

4. Solve the Dirichlet problem in the case of the picture.

(a) Formulate the partial differential equation anad the boundary values in accordance with the picture.

(b) Solve the equations by hand.

(c) Give a short description how this type of problems where treated during the course (i.e. describe the use of the program mylu2.cpp, in particular, the format of the input file.)



5. Write a program to solve the following system of differential equations with the initial value problem $y_1(1) = 3.0, y_2(1) = 1.0$

$$\frac{dy_1}{dx} = 10y_1(1.0 - y_2).$$

$$\frac{dy_2}{dx} = y_2(y_1 - 1.0).$$

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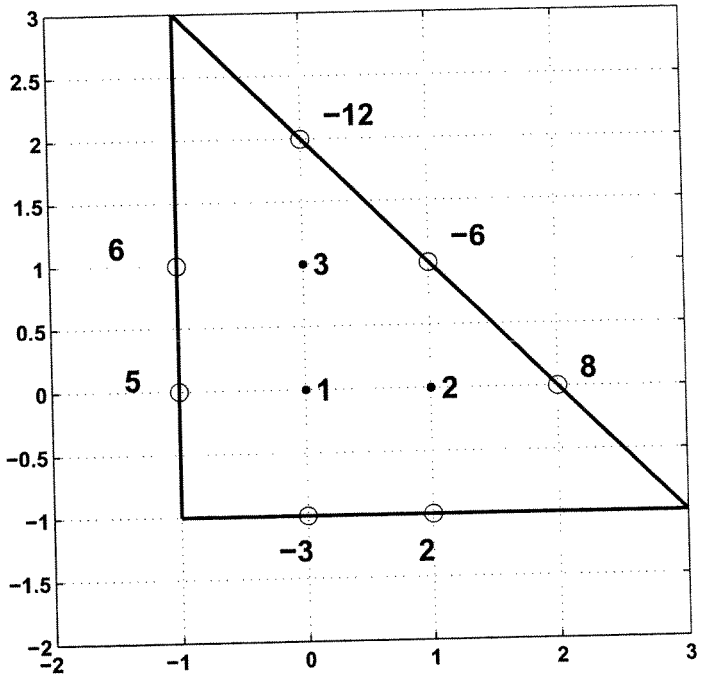

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