Department of mathematics and statistics
Model theory
Final exam
25.1.2007

Kysymykset suomeksi paperin kääntöpuolella.

1. Let L be a vocabulary and $\mathbb{K}$ the class of all finite L-structures. Show that $\mathbb{K}$ is not first-order axiomatizable.

2. Let $(A_i)_{i<\gamma}$ be an elementary chain of L-structures (i.e. $A_i \preccurlyeq A_j$ for each $i < j < \gamma$) and let $A = \bigcup_{i<\gamma} A_i$. Show that for each $j < \gamma$,
$$A_j \preccurlyeq A.$$

3. (a) Assume that A and B are countable and back-and-forth-equivalent L-structures. Show that A and B are isomorphic.
   (b) Write an example of a vocabulary L and two L-structures A and B of equal cardinality, which are back-and-forth equivalent but not isomorphic. No proof is needed.

4. Let L be a countable vocabulary and $\mathbb{K}$ a countable set of finitely generated L-structures. Assume that $\mathbb{K}$ has the Hereditary property (HP) and the Joint embedding property (JEP). Show that $\mathbb{K}$ is the age of some countable L-structure.

5. Let L be a countable first-order language and T an ω-categorical L-theory with infinite models. Show that for each $n < \omega$ and $p \in S_n(T)$, p is principal.