Department of Mathematics and Statistics
Logic I
Final Exam/Huuskonen
2007-03-06

1. Formulate a proposition that contains no other connectives than $\wedge$, $\neg$ and is logically equivalent to $(p_0 \vee p_1) \rightarrow (p_0 \rightarrow \neg p_1)$. Justify with a truth table.

2. Assume that the truth values of the claims "It is raining," "The wind is blowing" and "It is snowing" are definitely known. Nevertheless, the truth value of "If it is not snowing then it is raining and the wind is blowing" is ambiguous. What can you say about the weather? Justify with propositional logic. (Hint: there are two ways of placing parentheses in the formal expression.)

3. Prove by natural deduction that

$$\{\forall x_1(P_0(x_1) \leftrightarrow R_0(x_1, x_1)), \exists x_1 \forall x_0 R_0(x_0, x_1)\} \vdash \exists x_0 P_0(x_0).$$

4. Prove that if the $\forall$ quantifier in the previous problem is replaced with an $\exists$ quantifier, then the resulting problem cannot be solved. Symbolically,

$$\{\exists x_1(P_0(x_1) \leftrightarrow R_0(x_1, x_1)), \exists x_1 \forall x_0 R_0(x_0, x_1)\} \nvdash \exists x_0 P_0(x_0).$$

5. A stranger tells Mr. Virtanen that all red-haired people are lazy. Mr. Virtanen says, "I think that's not true." The stranger asks, "What? You're saying no red-haired people are lazy, huh?"

   Formalize in predicate logic the stranger's claim, Mr. Virtanen's denial and the stranger's interpretation of his denial. Are the last two formulas logically equivalent?

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2007-05-22

1. What is the conjunctive normal form of the proposition $(p_0 \vee p_1) \leftrightarrow (p_0 \wedge p_1)$? Justify with a truth table.

2. Present a tableau proof for the following sentence.

$$\forall x_1(P_0(x_1) \leftrightarrow R_0(x_1, x_1)) \rightarrow (P_0(c_0) \vee \neg\forall x_0 R_0(c_0, x_0))$$

3. Present a formal (so-called "Natural Deduction") proof for the following:

$$\{\forall x_1(P_0(x_1) \leftrightarrow R_0(x_1, x_1), \neg P_0(c_0)\} \vdash \neg\forall x_0 R_0(c_0, x_0).$$

4. Prove the following claim about the non-existence of a formal ("Natural Deduction") proof.

$$\{P_0(c_0) \vee \neg\forall x_0 R_0(c_0, x_0)\} \nvdash \forall x_1(P_0(x_1) \leftrightarrow R_0(x_1, x_1)).$$

   Justify by applying the Soundness Theorem appropriately.

5. Prove that the set $\{\wedge, \leftrightarrow\}$ is not a complete set of connectives.