Department of Mathematics and Statistics
Logic I
Examination 2 (kurssikoe)
May 9, 2008

1. Let the vocabulary be $L = \{R_0\}$. Let us define the L-structure $\mathcal{M}$ as follows:
$$\mathrm{dom}(\mathcal{M}) = \mathbf{N} = \{0, 1, 2, \ldots\}$$
and $R_0^{\mathcal{M}} = \{(n, m) \in \mathbf{N}^2 : m = 3n\}$. Show using the Tarski's definition of truth, that
$$\mathcal{M} \models \forall x_0 \exists x_1 R_0(x_0, x_1).$$

2. Construct a maximal semantic tree of the sentence
$$\exists x_0 \neg \forall x_1 \neg ((P_0(x_0) \to P_1(x_1)) \wedge (P_1(x_1) \to P_0(x_0))).$$
The tree will have an open branch that can be used to construct a model. Using this branch construct a model which satisfies the sentence. Can a model satisfy this sentence even if its domain has less elements than the structure which arose from the semantic tree?

3. Show by natural deduction that
$$\{\neg \exists x_0 \neg P_0(x_0)\} \vdash P_0(F_0^1(c_0)).$$
Hint: $\{\neg \exists x_0 \neg P_0(x_0)\} \vdash \forall x_0 P_0(x_0)$ and $\{\forall x_0 P_0(x_0)\} \vdash P_0(F_0^1(c_0))$.

4. One of the following properties of ball models is not definable. Prove that this particular property is not definable using the fact: If $\mathcal{M}$ and $\mathcal{N}$ are ball models with equal amount of black balls and both have at least n white balls, then $\mathcal{M}$ and $\mathcal{N}$ are n-equivalent.
   (a) If there are 99 black balls, then the amount of white balls is at least five times greater than the amount of black balls.
   (b) If there are five black balls, then the amount of white balls is divisible by five.
   (c) If there are five black balls, then the amount of white balls is different from the amount of black balls.