Department of Mathematics and Statistics
Logic I
Examination 1 (kurssikoe)
Feb 29, 2008

1. Construct a propositional sentence which is logically equivalent with the sentence $\neg(p_0 \wedge \neg p_1) \wedge (p_0 \vee \neg p_1)$ and is in

   (a) disjunctive normal form.

   (b) conjunctive normal form.

2. Consider the propositional sentence $A = \neg(p_0 \rightarrow p_1) \vee (\neg p_0 \vee p_1)$. Is A a tautology? In that case, construct a semantic proof (proof by semantic tree) for it. If A is not a tautology, then determine a truth distribution v such that $v(\neg A) = t$.

3. Construct a natural deduction that shows $\{A \vee B, \neg\neg\neg B\} \vdash A$.

4. Solve either (a) or (b) according to your choice:

   (a) Construct a natural deduction that shows that $A \vee \neg A$ is a tautology.

   (b) Let $M = \{1, 2, 3, 4\}$ and $L = \{c_0, c_1, P_0(x)\}$. We define a L-model $\mathcal{M}$ by putting $\mathrm{dom}(\mathcal{M}) = M$, $c_0^{\mathcal{M}} = c_1^{\mathcal{M}} = 3$, and $P_0^{\mathcal{M}} = \{1, 2\}$. Let $w : \{x_0, x_1, x_2, \ldots\} \rightarrow M$,

   $$w(x_n) = \begin{cases} 2, & \text{when } n \text{ is even} \\ 3, & \text{when } n \text{ is odd.} \end{cases}$$

   Which of the following claims are true and which are false? Short concise justifications are sufficient in this problem.

   (i) $\mathcal{M} \models_w P_0(x_{40})$
   (ii) $\mathcal{M} \models_w P_0(x_7)$
   (iii) $\mathcal{M} \models_w P_0(c_0)$
   (iv) $\mathcal{M} \models_{w(x_6/3)} \neg P_0(x_6)$
   (v) $\mathcal{M} \models_w \exists x_6 \neg P_0(x_6)$
   (vi) $\mathcal{M} \models_w \forall x_6 P_0(x_6)$