

1. Let $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation

$$L(\mathbf{x}) = \begin{bmatrix} -x_2 \\ 0 \end{bmatrix}, \text{ where } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2.$$

- (a) Determine whether the L is an isomorphism.
(b) Find $\text{Ker}(L)$ and $\text{Im}(L)$.
2. Let $P = (2, 2, 0)$, $Q = (0, 4, 1)$ and $R = (-1, 2, 3)$ be points of the space \mathbb{R}^3 .
(a) Find the area of the parallelogram determined by the vectors \overrightarrow{PQ} and \overrightarrow{PR} .
(b) Find the volume of the tetrahedron $OPQR$ (O denotes the origin).
3. A sales organization obtains the following data relating the number of salespersons to annual sales:

Number of salespersons	5	6	7	8	9	10
Annual sales (M€)	2,3	3,2	4,1	5,0	6,1	7,2

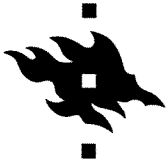
- (a) Present the given data as points in the (x, y) -coordinate system, where number of salespersons is in the x -axis and annual sales is in the y -axis.
(b) Find the least squares line that gives the best fit to the data.
(c) Estimate the annual sales when there are 14 salespersons.
4. Diagonalize orthogonally the symmetric matrix

$$A = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix},$$

this is, find a matrix P that orthogonally diagonalizes the matrix A , and determine $P^T A P$.

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1. Let $c \in \mathbb{R}$ and $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the mapping

$$L(\mathbf{x}) = \begin{bmatrix} x_2 \\ c + 1 \end{bmatrix}, \text{ where } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2.$$

- (a) Determine for which values of the constant c the mapping L is linear.
(b) Find $\text{Ker}(L)$ and $\text{Im}(L)$ whenever L is linear.
2. Let $P = (1, 4, -1)$, $Q = (0, 2, 1)$ and $R = (-1, 1, 2)$ be points of the space \mathbb{R}^3 .
(a) Find the area of the triangle PQR .
(b) Find the volume of the parallelepiped determined by the position vectors of P , Q and R .

3. Let

$$S = ([1 \ 1 \ 1]^T, [-1 \ 1 \ 0]^T, [1 \ 2 \ 1]^T)$$

be a basis of \mathbb{R}^3 .

- (a) Use the Gram–Schmidt process to transform the basis S into an orthonormal basis S_0 .
(b) Express the vectors of the basis S as linear combinations of the vectors of the orthonormal basis S_0 .
4. Determine A^{100} , when $A = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$.

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