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HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

Department of Mathematics and Statistics
Linear algebra and matrices II
Course examination 14.12.2005

1. Let $L: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation
$$L(\mathbf{x}) = \begin{bmatrix} -x_2 \\ 0 \end{bmatrix}, \text{ where } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2.$$
   (a) Determine whether the L is an isomorphism.
   (b) Find $\text{Ker}(L)$ and $\text{Im}(L)$.

2. Let $P = (2,2,0)$, $Q = (0,4,1)$ and $R = (-1,2,3)$ be points of the space $\mathbb{R}^3$.
   (a) Find the area of the parallelogram determined by the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.
   (b) Find the volume of the tetrahedron $OPQR$ (O denotes the origin).

3. A sales organization obtains the following data relating the number of salespersons to annual sales:

Number of salespersons	5	6	7	8	9	10
Annual sales (M€)	2,3	3,2	4,1	5,0	6,1	7,2

   (a) Present the given data as points in the (x,y)-coordinate system, where number of salespersons is in the x-axis and annual sales is in the y-axis.
   (b) Find the least squares line that gives the best fit to the data.
   (c) Estimate the annual sales when there are 14 salespersons.

4. Diagonalize orthogonally the symmetric matrix
$$A = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix},$$
this is, find a matrix P that orthogonally diagonalizes the matrix A, and determine P^TAP.

Please answer to the course questionnaire

http://mathstat.helsinki.fi/kurssit/kysely/index.en.html

right after the examination!

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Department of Mathematics and Statistics
Linear algebra and matrices II
Course examination (extra) 21.12.2005

1. Let $c \in \mathbb{R}$ and $L\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the mapping
$$L(\mathbf{x}) = \begin{bmatrix} x_2 \\ c+1 \end{bmatrix}, \text{ where } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2.$$
   (a) Determine for which values of the constant c the mapping L is linear.
   (b) Find Ker(L) and Im(L) whenever L is linear.

2. Let $P = (1, 4, -1)$, $Q = (0, 2, 1)$ and $R = (-1, 1, 2)$ be points of the space $\mathbb{R}^3$.
   (a) Find the area of the triangle PQR.
   (b) Find the volume of the parallelepiped determined by the position vectors of P, Q and R.

3. Let
$$S = ([1\ \ 1\ \ 1]^T, [-1\ \ 1\ \ 0]^T, [1\ \ 2\ \ 1]^T)$$
be a basis of $\mathbb{R}^3$.
   (a) Use the Gram–Schmidt process to transform the basis S into an orthonormal basis S_0.
   (b) Express the vectors of the basis S as linear combinations of the vectors of the orthonormal basis S_0.

4. Determine A^{100}, when $A = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$.

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