LINEAARIALGEBRA JA MATRIISILASK.
9.8.2007

1. What is the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 4 & 3 \end{bmatrix}$$

2. Assume that A and B are invertible matrices. Show that A^{-1} and AB are also invertible. What about $A+B$? Is it necessarily also invertible?

3. For the matrix A

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$$

find a basis for it's nullspace. Additionally determine the rank of A.

4. Assume that $v_1, \dots, v_k$ are linearly dependent vectors in a vector space V. Show that

$$\dim \operatorname{span}(v_1, \dots, v_k) < k$$

5. Assume that $\{e_1, \dots, e_n\}$ is a basis in $\mathbb{R}^n$. Show that $\{f_1, \dots, f_n\}$ is also basis in $\mathbb{R}^n$, when

$$f_i = e_1 + e_2 + \dots + e_j \; ; \; j = 1, \dots, n.$$

Linear algebra and matrix theory I

Final exam 13.5.2008

1. Let

$$A = \begin{pmatrix} 1 & a & 0 \\ a & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix},$$

where $a \in \mathbb{R}$. Let also $\lambda \in \mathbb{R}$. Determine all possible values of constants a and λ such that the equation

$$AX = \begin{pmatrix} 1 \\ \lambda \\ \lambda^2 \end{pmatrix}$$

has a solution $X = (x_1, x_2, x_3)^t$. **Hint**: First study the case when the matrix A is regular.

2. Do the sequences

   (a) $(1, x+1, (x+1)^2)$,

   (b) $(x, x^2-1, (x+1)^2)$,

   (c) $(x+1, x^2-1)$,

   span the vector space P_2 of polynomials of order ≤ 2.

3. Determine bases for the row–, column– and null–spaces of the matrix

$$B = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{pmatrix}.$$

4. Let P_2 be the vector space of polynomials of order ≤ 2. Equip it with the basis $\mathbf{e} = (1, x, x^2)$. Consider the map (the derivative map)

$$d : P_2 \to P_2, \quad p \mapsto p',$$

where p' is the derivative of the polynomial p. Determine the matrix of d with respect to basis $\mathbf{e}$. Is d one–to–one? Is it onto?

5. Assume that A and B are real $n \times n$–matrices, which commute, i.e. $AB = BA$, and asume that AB is regular. Show that also A and B are regular.

Department of Mathematics and Statistics
Linear algebra and matrixes I (Lineaarialgebra ja matriisilaskenta I)
Exam 3.3.2009
Teacher: Petteri Harjulehto

1. Let

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & -6 \\ -2 & 3 \end{bmatrix}$$

be matrixes in $\mathbb{R}^{2\times 2}$. Does the equation $AB = BA$ hold?

2. Solve the linear system

$$\begin{cases} \phantom{2x_1 +{}} 2x_2 + 3x_3 - 4x_4 = 1 \\ \phantom{2x_1 + 2x_2 +{}} 2x_3 + 3x_4 = 4 \\ 2x_1 + 2x_2 - 5x_3 + 2x_4 = 4 \\ 2x_1 \phantom{{}+2x_2} - 6x_3 + 9x_4 = 7 \end{cases}$$

by Gauss or Gauss-Jordan elimination.

3. Define the term "invertible matrix". What is the inverse matrix of

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}?$$

4. Let V be a linear space and let $(\bar{v}_1, \bar{v}_2, \bar{v}_3)$ be a linearly independent sequence in V. We define $\bar{w}_1 = \bar{v}_1$, $\bar{w}_2 = \bar{v}_1 - \bar{v}_2$ and $\bar{w}_3 = \bar{v}_1 - \bar{v}_2 - \bar{v}_3$. Is the sequence $(\bar{w}_1, \bar{w}_2, \bar{w}_3)$ linearly independent in V?

5. Define the term "basis". Give an example
(a) of a basis for $\mathbb{R}^3$ that contains the vector $[1, 1, 1]^T$;
(b) of a linearly independent sequence that is not a basis for $\mathbb{R}^3$.