Life insurance mathematics 19.12.2006

1. Let i be a given annual effective interest rate and $i^{(m)}$ the corresponding nominal interest rate, convertible m times per year ($m \in \mathbf{N}$). Show that $i^{(m)}$ is decreasing in m.

2. Define the mortality of (x) at the age $x+t$. Given a continuous mortality μ_x, derive an expression for ${}_tp_x$.

3. Consider two independent lives. The mortality of life 1 is μ and the mortality of life 2 is $c\mu$ where $c > 1$ is a constant. The current ages are both zero. Calculate the probability that the remaining lifetime of life 2 exceeds that of life 1.

4. Consider a contract where the company pays S euros to the insured at the beginning of each year k if the insured is alive, $k = n, \ldots, 2n$, $n \in \mathbf{N}$. The insured pays the annual premium P at the beginning of each year k if he is alive, $k = 0, \ldots, n-1$. The premium is determined by the principle of equivalence. The mortality μ and the force of interest δ are supposed to be positive constants.

a) Determine P in terms of S, μ and δ.

b) At time $t = 1$, just before the second premium payment, the insured asks the company to change the benefit S such that no premiums will be paid in the future. Determine the new benefit.

5. Consider a whole life insurance where the payment S_k takes place at the end of the year k in case the insured dies durig the year k, $k = 0, 1, \ldots, n-1$. The insured pays the amount P_k at the beginning of the year k, $k = 0, 1, \ldots, n-1$, if he is then alive. The insured is of age x at the beginning of the contract. Let δ be the (constant) force of interest. Let further $V(k)$ be the net premium reserve of a living insured just before he pays the kth premium P_k. Show that

$$e^{\delta}(V(k) + P_k) = q_{x+k}S_k + p_{x+k}V(k+1), \quad k = 0, 1, \ldots, n-2.$$

Show that the formula also holds for $k = n-1$ with convention $V(n) = 0$.

Life insurance mathematics 12.4.2007

1. Let i be a given annual effective interest rate and $n \in \mathbf{N}$. At time k, a saving $B(k)$ to a bank account takes place, $k = 0, 1, \ldots, n$. Denote by $V(k)$ the amount at the account at time k just before the saving $B(k)$ takes place (the interest is compounded in yearly basis).

Let $\alpha > i$ and $C > 0$. Determine $\{B(k); k = 0, 1, \ldots, n-1\}$ such that $V(k) = C(1+\alpha)^k$, $k = 1, 2, \ldots, n$.

2. Define the mortality of (x) at the age $x + t$. Given a continuous mortality μ_x, derive an expression for ${}_tp_x$.

3. Consider two causes of the death with corresponding independent life times T_1 and T_2. Given that i would be the only cause in force, the future life time would have the distribution F_i,

$$F_i(t) = 1 - e^{-\mu_i t},\ i = 1, 2,$$

where μ_1 and μ_2 are positive constants and $t \geq 0$ (the real life time is $\min(T_1, T_2)$). Consider a contract where the company pays S euros to the insured at the beginning of each year k if the insured is alive, $k = 1, \ldots, n$, $n \in \mathbf{N}$. The insured pays the whole premium at the beginning of the contract. Determine that premium.

4. Consider a contract where the company pays S euros to the insured at the end of the year $2n$ if th insurd is alive at that time. The insured pays the annual premium P at the beginning of each year k if he is alive, $k = 1, \ldots, 2n$. The premium is determined by the principle of equivalence. The mortality μ and the force of interest δ are supposed to be positive constants.

a) Determine P in terms of S, μ and δ.

b) At the end of the year n, just before the premium payment, the insured asks the company to change the benefit S such that no premiums will be paid in the future. Determine the new benefit.

5. Consider a whole life insurance where the payment S_k takes place at the end of the year k in case the insured dies durig the year k, $k = 0, 1, \ldots, n-1$. The insured pays the amount P_k at the beginning of the year k, $k = 0, 1, \ldots, n-1$, if he is then alive. The insured is of age x at the beginning of the contract. Let δ be the (constant) force of interest. Let further $V(k)$ be the net premium reserve of a living insured just before he pays the kth premium P_k. Show that

$$e^{\delta}(V(k) + P_k) = q_{x+k}S_k + p_{x+k}V(k+1), \quad k = 0, 1, \ldots, n-2.$$

Show that the formula also holds for $k = n-1$ with convention $V(n) = 0$.