LIE ALGEBRAS

~~Examination January 26th, 2006.~~

14. 8. 08

Solve 5 problems out of 6!

1. Find all two dimensional Lie algebras, up to isomorphism.

2. Consider the 3-dimensional complex Lie algebra with basis a, b, h and commutation relations

$$[a, b] = -a, \ [h, a] = [h, b] = 0.$$

Construct a faithful 2-dimensional representation of this Lie algebra and find the weights of the representation.

3. Using the root axioms, find the possible values for the angle between a pair of roots.

4. Let $h_1 = e_{11} - e_{22}$ and $h_2 = e_{22} - e_{33}$ be the basis vectors for the Cartan subalgebra of the simple Lie algebra A_2. Here e_{ij} are the standard Weyl basis vectors. What are the posssible values $\lambda_i = \lambda(h_i)$ when λ is an integral dominant weight?

5. Let V_λ be an irreducible module with highest weight λ for the affine Kac–Moody algebra constructed from the simple Lie algebra $\mathbf{sl}(2)$. Assume the hermiticity relations $(T_a^n)^* = -T_a^n$ for the standard basis of the loop algebra. Derive from this the inequality $\lambda(h) \leq k$. The commutation relations are

$$[T_a^n, T_b^m] = \lambda_{ab}^c T_c^{n+m} + kn\delta_{n,-m}\mathrm{tr}(T_a T_b),$$

where the T_a's are the antihermitean 2×2 matrices defining the Lie algebra $\mathbf{sl}(2)$.

6. Construct an example of a commutative but not cocommutative Hopf algebra.