Kvasisäännölliset kuvaukset

Tentti 21.12. 2000

Matematiikan laitos

Helsingin yliopisto

VASTAA NELJÄÄN TEHTÄVÄÄN (SUOMEKSI TAI ENGLANNIKSI)!

1. This problem deals with the Möbius transformations of $GM(\mathbf{B}^n)$.

(a) Given $s \in (0,1), f \in GM(\mathbf{B}^n)$, find an upper bound for the Lipschitz constant $\mathrm{Lip}(f|B^n(s))$.

(b) Let $a \in \mathbf{B}^n$. Give the definition of the map $T_a \in GM(\mathbf{B}^n)$. Is it true that $T_a^{-1} = T_{-a}$?

(c) What, in your opinion, is the most important property of Möbius transformations?

2. (a) Give the definition of the hyperbolic metric of $\mathbf{B}^n$. Do the following inequalities hold for $x, y \in \mathbf{B}^n$? Give reasons for your answer.

$$(a1) \quad \rho_{\mathbf{B}^n}(|x|e_1, |y|e_1) \le \rho_{\mathbf{B}^n}(x,y),$$

$$(a2) \quad \rho_{\mathbf{B}^n}(x,y) \le \rho_{\mathbf{B}^n}(-|x|e_1, |y|e_1).$$

(b) Give the definition of the quasihyperbolic metric of a domain in $\mathbf{R}^n$. Let $0 < a < b < \infty, R = \mathbf{B}^n(b)\backslash\overline{\mathbf{B}^n}(a)$, $t \in (a,b)$. Find an upper bound for the quasihyperbolic diameter $k_R(C), C = S^{n-1}(t)$.

3. Let $t > r > s > 0$, $E \subset \mathbf{B}^n(s)$ and $\Delta_a = \Delta(E, S^{n-1}(a))$. Show that $\mathsf{M}(\Delta_r) \le c\mathsf{M}(\Delta_t)$, where c is only dependent on n, r, s and t.

4. Let $G, G' \subset \mathbf{R}^n$ and $f : (G, k_G) \to (G', k_{G'})$ be uniformly continuous, and let $b' \in \partial G'$. Show that $u : G \to R_+, u(x) = |f(x) - b'|$ satisfies Harnack's inequality.

5. (a) Formulate the definitions of the conformal invariants μ_G and λ_G and their transformation rules under quasiconformal maps.

(b) Formulate the Schwarz lemma for quasiregular maps of $\mathbf{B}^n$ into $\mathbf{B}^n$.

6. Consider two squares $Q(a), Q(b)$ in the plane $\mathbf{R}^2$ with sides parallel to the coordinate axes and side lengths $2a, 2b$, $0 < a < b$, resp. Suppose that the center