



1. Suppose that in a childbirth
 - the probability of a girl is $\frac{1}{2}$,
 - identical twins have the same gender,
 - genders of unidentical twins are independent,
 - $\frac{1}{4}$ of all twins are identical.
 - a) Find the probability that in a twin birth the both children are girls.
 - b) In a twin birth both children turn out to be girls. What is the probability that the twins are identical?

2. A mathematics course consists of 24 lectures during a term (2 lectures weekly for 12 weeks). The course is taken part by 20 students, of which 16 are female and 4 are male. They all come to every lecture.
 - a) In the beginning of every lecture one student is chosen randomly to write notes of the lecture. Let a random variable X represent the number of male students chosen during the term. What is the distribution of X ? Find $P(X \geq 2)$.
 - b) In the end of the term it is chosen randomly a group of four students to check and to put the notes written during the course together. Let a random variable Y represent the number of female students chosen to the group. What is the distribution of Y ? Find the probability that there are as many women as men in the group.

3. The number of persons coming to a statistics course is a Poisson-distributed random variable with the expectation 50. There are 60 seats in the lecture room. By using the normal approximation find the probability that more than 60 persons come to the course, and the lecture room has to be changed to a bigger one. *Reminder:* If $X \sim \text{Poisson}(\lambda)$, then $EX = D^2X = \lambda$.

Choose *only one* of the following problems 4A and 4B to which you answer. If you return a sheet of paper including solutions to the both problems 4A and 4B, and you don't indicate which one of the solutions should be marked, then the solution giving less points will be regarded.

4A. Let A be an event of a probability space (Ω, \mathcal{F}, P) . Find out if it is possible that A is independent of itself, this is, $A \perp A$.

4B. Let a function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$,

$$f(x, y) = \begin{cases} 3x, & 0 < x < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Show that f defines a density function of a continuous distribution. Suppose that f is the density function of a continuous random vector (X, Y) . Determine the marginal distribution function f_X of X and find EX and D^2X .

Please answer to the course questionnaire

<http://mathstat.helsinki.fi/kurssit/kysely/index.en.html>

right after the examination!



1. One coin in a collection of 65 in a purse has two heads. The rest are fair with head and tail. A coin is chosen at random from the purse and then tossed. If head turns up six times in a row, what is the probability that the chosen coin is the two-headed one?
2. Suppose that the time T (in hours) required to repair a car is an exponentially distributed random variable. From long experience it is known in a garage that it takes on the average 2 hours to repair a car. Hence you may suppose that $ET = 2$. In a morning mister K leaves his car to be repaired and repairing it is started at 8.
 - (a) Find the probability that the car is already repaired at 12 when mister K calls to the garage.
 - (b) Mister K finds out at 12 when he calls to the garage that the car is not yet repaired. With this condition find the probability that repairing of the car is still unfinished when the working day ends at 16.
3. A die is rolled 420 times. Using the normal approximation find an approximated value for the probability that the sum of the rolls lies between 1400 and 1550.

4. (a) Find a constant $c \in \mathbb{R}$ such that the function $f: \mathbb{R} \rightarrow \mathbb{R}$,

$$f(x) = \begin{cases} c|x|, & -2 \leq x \leq 2, \\ 0, & \text{otherwise,} \end{cases}$$

defines a density function.

- (b) Let f be the density function of a random variable X . Determine the cumulative distribution function F of X .
- (c) Find $P(X^2 > 2)$.

Taulukko 1. Standardinormaalijakauman kertymäfunktion Φ arvoja, $\Phi(x) =$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}t^2} dt.$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.500000	.503989	.507978	.511966	.515953	.519938	.523922	.527903	.531881	.535856
0.1	.539828	.543795	.547758	.551717	.555670	.559618	.563560	.567495	.571424	.575345
0.2	.579260	.583166	.587064	.590954	.594835	.598706	.602568	.606420	.610261	.614092
0.3	.617911	.621720	.625616	.629300	.633072	.636831	.640576	.644309	.648027	.651732
0.4	.655422	.659097	.662757	.666402	.670031	.673645	.677242	.680822	.684386	.687933
0.5	.691462	.694974	.698468	.702944	.705402	.708840	.712260	.715661	.719043	.722405
0.6	.725747	.729069	.732371	.735653	.738914	.742154	.745373	.748571	.751748	.754903
0.7	.758036	.761148	.764238	.767305	.770350	.773373	.776373	.779350	.782305	.785236
0.8	.788145	.791030	.793892	.796731	.799546	.802338	.805106	.807850	.810570	.813267
0.9	.815940	.818589	.821214	.823814	.826391	.828944	.831472	.833977	.836457	.838913
1.0	.841345	.843752	.846136	.848495	.850830	.853141	.855428	.857690	.859929	.862143
1.1	.864334	.866500	.868643	.870762	.872857	.874928	.876976	.879000	.881000	.882977
1.2	.884930	.886861	.888768	.890651	.892512	.894350	.896165	.897958	.899727	.901475
1.3	.903200	.904902	.906582	.908241	.909877	.911492	.913085	.914656	.916207	.917736
1.4	.919243	.920730	.922196	.923642	.925066	.926471	.927855	.929219	.930563	.931889
1.5	.933193	.934478	.935744	.936992	.938220	.939429	.940620	.941792	.942947	.944083
1.6	.945201	.946301	.947384	.948449	.949497	.950528	.951543	.952540	.953521	.954486
1.7	.955434	.956367	.957284	.958185	.959070	.959941	.960796	.961636	.962462	.963273
1.8	.964070	.964852	.965620	.966375	.967116	.967843	.968557	.969258	.969946	.970621
1.9	.971283	.971933	.972571	.973197	.973810	.974412	.975002	.975581	.976148	.976704
2.0	.977250	.977784	.978308	.978822	.979325	.979818	.980301	.980774	.981237	.981691
2.1	.982136	.982571	.982997	.983414	.983823	.984222	.984614	.984997	.985371	.985738
2.2	.986097	.986447	.986791	.987126	.987454	.987776	.988089	.988396	.988696	.988989
2.3	.989276	.989556	.989830	.990097	.990358	.990613	.990862	.991106	.991344	.991576
2.4	.991802	.992024	.992240	.992451	.992656	.992857	.993053	.993244	.993431	.993613
2.5	.993790	.993963	.994132	.994297	.994457	.994614	.994766	.994915	.995060	.995201
2.6	.995339	.995473	.995604	.995731	.995855	.995975	.996093	.996207	.996319	.996427
2.7	.996533	.996636	.996736	.996833	.996928	.997020	.997110	.997197	.997282	.997365
2.8	.997445	.997523	.997599	.997673	.997744	.997814	.997882	.997948	.998012	.998074
2.9	.998134	.998193	.998250	.998305	.998359	.998411	.998462	.998511	.998559	.998605
3.0	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	.998650	.999032	.999313	.999517	.999663	.999767	.999841	.999892	.999928	.999952