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Department of Mathematics and Statistics
Introduction to probability
Course examination 5.3.2007

1. Mr. K has three, four or five different pairs of socks in his wardrobe, three with probability $\frac{1}{3}$, four with probability $\frac{7}{15}$ and five with probability $\frac{1}{5}$.
   a) In a dark morning K chooses randomly socks for his both feet. What is the probability that the socks are a pair?
   b) Later in the same day K observes that the socks that he chose in the morning are not a pair. What is the probability with this condition that there were five pairs of socks in the wardrobe?

2. Service life of a pair of skis in use is exponentially distributed with the expected value 5 years. Mrs. Strong got a new pair of skis in Christmas and she started to use them in the beginning of the year 2007. Find the probability that she uses those skis
   a) after the year 2010,
   b) after the year 2010 with the condition that she uses the skis in the beginning of the year 2010.

3. Statistical probability for a born baby to be a boy is 0,512. In a city 1000 babies are born annually. By using the normal approximation find the probability that the number of boys exceeds the number of girls.

Choose *only one* of the following problems 4A and 4B to which you answer. If you return a sheet of paper including solutions to the both problems 4A and 4B, and you don't indicate which one of the solutions should be marked, then the solution giving less points will be regarded.

4A. Let A and B_1 and B_2 be such events of a probability space $(\Omega, \mathcal{F}, P)$ that $A \perp\!\!\!\perp B_i$, $i = 1, 2$. Show that if $B_1 \cap B_2 = \emptyset$, then $A \perp\!\!\!\perp (B_1 \cup B_2)$.

4B. Let a function $f\colon \mathbb{R}^2 \to \mathbb{R}$ be given as

$$f(x,y) = \begin{cases} 2x^{-3}e^{-y}, & x > 1,\ y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Show that f defines a density function of a continuous distribution. Suppose that f is the density function of a random vector (X, Y). Are X and Y independent? Are they uncorrelated?

Please answer to the course questionnaire

`http://mathstat.helsinki.fi/kurssit/kysely/index.en.html`

right after the examination!

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Introduction to probability
Final examination 12.4.2007

1. Four coins are tossed five times. Let X be the number of heads for the first toss and let Y be the total number of heads for five tosses.
   a) What are the distributions and the expectations of X and Y?
   b) Are X and Y independent?
   c) Find $P(X \leqslant 2)$ and $P(Y \geqslant 19 \mid X \leqslant 2)$.

2. A hitchhiker knows that on the average 50 cars pass before the first car stops. Suppose that the drivers make decisions to stop independently. Find the probability that already the first car arriving stops.

3. Total price of purchases is estimated by the closest 5 cents sharp. The estimation error of the total price of one customers purchases causes a loss to the shopkeeper. This is a random variable that takes values $-2, -1, 0, 1, 2$, each of them with probability $\frac{1}{5}$. Let X be the total loss caused by 10 000 customers. Find the probability $P(X > 2\,€)$ by using the normal approximation.

4. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$,
$$f(x,y) = \begin{cases} 6y, & 0 < x < x+y < 1, \\ 0, & \text{otherwise}, \end{cases}$$
be the density function of a continuously distributed random vector (X, Y). Find the correlation coefficient $\mathrm{Corr}(X, Y)$.

5. a) Show the *lost memory property* of the exponential distribution: If $X \sim \mathrm{Exp}(\lambda)$ for some $\lambda > 0$, then
$$P(X > t + h \mid X > t) = P(X > h) \text{ for all } t \geqslant 0,\ h > 0.$$
   b) It is known that a random variable X is exponentially distributed. Find the parameter λ of the distribution if
$$P(X > 10 \mid X > 6) = \frac{1}{e}.$$