Final exam - loppukoe

Introduction to generalized quantifiers II

19.5.2005

1. Define the union of an elementary chain $(\mathcal{M}_n, Q_n)$, $n \in \mathbb{N}$, of weak models and prove that the union is an elementary extension of each member of the chain.
2. Show that every uncountable weak model $(\mathcal{M}, \exists^{\geq \aleph_1})$ satisfies Keisler's axiom KA.
3. Suppose $(\mathcal{M}, Q)$ is a countable weak cofinality model and $\varphi(x, y)$ is a formula of $L_{\omega\omega}(\mathcal{Q})$ such that $(\mathcal{M}^*, Q) \models \mathcal{Q}^* xy\varphi(x, y)$. Let c be a new constant symbol and T the theory

$$\{\theta : (\mathcal{M}^*, Q) \models \theta\} \cup \{\varphi(a, c) : a \in M\}.$$

   Prove that $T \cup \{\theta(c)\}$ is consistent if and only if $(\mathcal{M}^*, Q) \models \forall x \exists y(\varphi(x, y) \wedge \theta(y))$.
4. Prove that the model $(\mathbb{N} + \mathbb{Z} + \mathbb{Z}, <)$ is not $\aleph_0$-homogeneous.

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15.6.2005, Tapio Eerola

1. Show that player I has a winning strategy in the monotone model existence game $\mathrm{MEG}^{\mathcal{Q}}(T, L)$, where L consists of unary predicate symbols P and R and a T is $\{\check{\mathcal{Q}}x(\neg Px \wedge \neg Rx), \mathcal{Q}xPx \vee \mathcal{Q}xRx\}$.
2. Suppose $(\mathcal{M}, Q)$ is a non-trivial countable monotone ideal weak model and $\varphi(x)$ is a formula of $L_{\omega\omega}(\mathcal{Q})$ such that $(\mathcal{M}^*, Q) \models \mathcal{Q}x\varphi(x)$. Let c be a new constant symbol and T the theory

   $$\{\theta : (\mathcal{M}^*, Q) \models \theta\} \cup \{\varphi(c)\} \cup \{\neg\psi(c) : (\mathcal{M}^*, Q) \models \neg\mathcal{Q}x\psi(x)\}.$$

   Prove that $T \cup \{\psi(c)\}$ is consistent if and only if $(\mathcal{M}^*, Q) \models \mathcal{Q}x(\varphi(x) \wedge \psi(x))$.
3. Describe the proof of the Completeness Theorem of $\exists^{\geq\aleph_1}$ using the Main Lemma and the Precise Extension Lemma. You need not prove the two lemmas.
4. Prove that the model $(\mathbb{N}+\mathbb{Z}, <)$ is $\aleph_0$-homogeneous.

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Introduction to generalized quantifiers II

10.8.2005, Jarmo Kontinen

1. Suppose $\mathcal{M}$ is an L-structure and Q and Q' are weak quantifiers on M such that

$$\mathrm{Def}(\mathcal{M}, Q) \cap Q = \mathrm{Def}(\mathcal{M}, Q) \cap Q'.$$

   Show that for all $\varphi \in L_{\omega\omega}(\mathcal{Q})$ and all assignments s

$$(\mathcal{M}, Q) \models_s \varphi \iff (\mathcal{M}, Q') \models_s \varphi.$$

2. Suppose L is a countable vocabulary and T is a set of L-sentences of $L_{\omega\omega}(\mathcal{Q})$. Show that if T has a monotone model $(\mathcal{M}, Q)$, then Player II has a winning strategy in $\mathrm{MEG}^{\mathcal{Q}}(T, L)$.
3. Prove the Union Lemma for weak models: The union of an elementary chain $(\mathcal{M}_n, Q_n)$ $(n \in \mathbb{N})$ of weak models is an elementary extension of each element of the chain.
4. Prove that if $\mathcal{M}$ and $\mathcal{N}$ are $\aleph_0$-homogeneous. satisfy the same n-types for all n, and $\mathcal{M} \equiv \mathcal{N}$, then $\mathcal{M} \simeq_p \mathcal{N}$

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Introduction to generalized quantifiers II

25.10.2005

1. Suppose L is a vocabulary, $\mathcal{M}$ an L-structure and $X \subseteq M$ such that for all L-formulas $\varphi(x_0, \ldots, x_n)$ the following holds:

   If $a_0, \ldots, a_{n-1} \in X$ and $\mathcal{M} \models \varphi(a_0, \ldots, a_{n-1}, a_n)$ for some $a_n \in M$, then $\mathcal{M} \models \varphi(a_0, \ldots, a_{n-1}, a'_n)$ for some $a'_n \in X$.

   Show that $\mathcal{M}^{(X)} \prec \mathcal{M}$.
2. Suppose L is a vocabulary, $\mathcal{M}$ an L-structure and $\mathcal{F}$ a family of functions such that every L-formula has a Skolem-function $\in \mathcal{F}$ in $\mathcal{M}$. Suppose $M_0 \subseteq M$ is closed under all functions in $\mathcal{F}$. Then M_0 is the universe of an elementary submodel $\mathcal{M}_0$ of $\mathcal{M}$.
3. Give a winning strategy for player II in the monotone model existence game $\mathrm{MEG}^{\mathcal{Q}}(T, L)$, where L consists of unary predicate symbols P and R and a T is $\{\check{\mathcal{Q}}x(\neg Px \vee \neg Rx), \mathcal{Q}xPx \wedge \mathcal{Q}xRx\}$.
4. Show that if $\varphi(x)$ is countable-like in a plural non-trivial $(\mathcal{M}, Q)$, then $(\mathcal{M}, Q) \models \neg \mathcal{Q}x\psi(x)$.
5. Suppose $R \in Q^{\mathrm{cf}}_{\omega}(M)$ and $S \in Q^{\mathrm{cf}}_{>\omega}(M)$. Show that there is no order preserving mapping $f : (M, <_S) \to (M, <_R)$ (i.e. $x <_S y \to f(x) <_R f(y)$) whose range is cofinal in $<_S$.